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On the so-called "Grenzfläche" in the Current due to the Difference of Density.

By

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ABSTRACT.

What difference in height of sea-level should be produced by a local difference in the density of sea water? Or, what ratio must the difference in level in the dynamical equilibrium hold to that when statical equilibrium is assumed? This ratio must obviously have an intimate connection with the "Grenzfläche" which is defined as an equipressure layer lying horizontal in the water.

The present paper treats theoretically the Grenzfläche in a basin of rectangular section, in which the density of the water differs from place to place in the upper part of some thickness, but in the remaining lower part it is homogeneous throughout.

According to the result, if local inequality of density extend equally throughout the whole depth from surface to bottom, the real level difference will be only $3/8$ to $4/8$ of the statical one. But the thinner the part in which inequality of water exists, the nearer the level difference in dynamical equilibrium approaches to that in statical equilibrium; and when the part of heterogeneous water is, as in the actual ocean basin, less than about $1/10$ of the whole depth, the sea surface will assume practically the same level as if it were in statical equilibrium.

Moreover, the calculation shows that the no-current layer does not always coincide with the Grenzfläche of constant pressure, but will exist generally somewhat lower.

By the bye, the paper involves a discussion on the inequality of the pressure upon the sea bottom which will be caused by the difference in density of water, because the bottom-pressure may have some relation to the frequency of submarine earthquakes, as pointed out by Dr. Omori.

Introduction.

The oceanography teaches us that the temperature and the salinity, and, consequently, the density, of the water in the ocean surface greatly differs according to locality, and that these local differences become rapidly less and less as the depth increases, until below a level of some hundreds of meters they are very small, and the water may be regarded

as practically uniform throughout the whole ocean. Thus, although the differences in the density of sea water are limited only in the upper stratum which is very thin compared with the mean depth (about 4000 meters) of the world ocean, sea water can not hold a statical equilibrium. In a deep level, the pressure in places where the water is denser must be greater than that where the upper water is lighter, and so water will flow from the former regions towards the latter, with the result that the water surface in the latter regions rises up higher than in the former. As soon as an inclination of surface is produced, the distribution of pressure in a horizontal section of the uppermost strata becomes contrary to that in the bottom layer; i. e., the pressure in regions of a higher level of surface will be greater than that where the surface level is lower, and therefore the water in the upper layers must flow from the higher level regions to the lower in order to diminish the level difference of the surface. Thus, in the upper and lower parts of the sea there must generate two systems of currents and of pressure distributions, which are reverse to each other. Between these two systems somewhere one may conceive an equipressure layer which is horizontal, and also another layer in which no current flows. H. Mohn¹ assumed these two layers—equipressure and no-current—to be coincident and called the layer the *Grenzfläche*. Krümmel² states that for a basin of rectangular section the *Grenzfläche* will lie at half the depth of the bottom.

Now the prime object of the present paper is to throw a light on the problem:

(i) What difference of sea level will be produced by a local difference in the density of water when dynamical equilibrium has been attained?

But this is almost the same thing as determining

(ii) the depth of the *Grenzfläche*.

Here I define a *Grenzfläche* as such a one in which the horizontal gradient of pressure is nil.

For me it is not self-evident but to be discussed

(iii) Whether the *Grenzfläche* must always coincide with the no-current layer, or not.

Moreover I will, by the way, discuss the question:

(iv) What difference of pressure at the sea-bottom will be caused by the difference in the density of water?

¹ M. P. Rudzki, *Physik der Erde*, 447 (1911).

² *Handbuch der Ozeanographie*, II, 482 (1911).

For, Dr. Omori¹ investigated the frequency of earthquakes of suboceanic origin with relation to the pressure on the sea-bottom. He assumed that a seasonal fluctuation of mean sea-level, the barometric effect being removed, must produce a variation in the pressure on the sea-bottom, directly proportional to the difference in height of the surface level. But Dr. Nagaoka² disagreed with the opinion of Omori, and said :

"If the necessary temperature correction be introduced, it would follow that there are slight irregularities which would have no relation to seismic frequency."

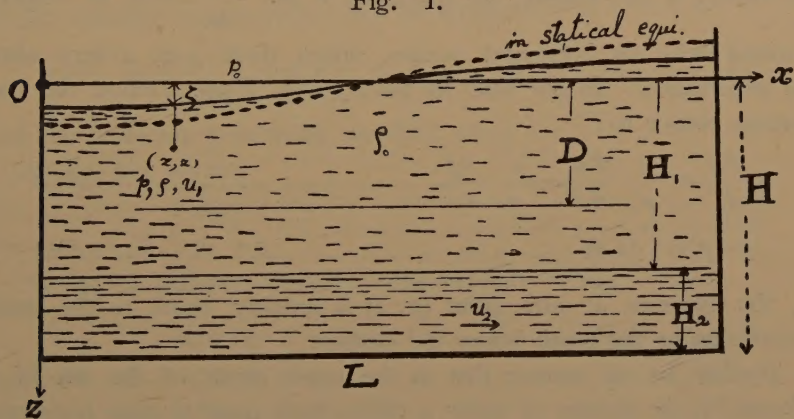
This statement seems to involve that the difference in the density of water would not affect the pressure on the sea-bottom.

The present writer is of a somewhat different opinion, which will be seen at the end of this paper.

I. Fundamental Equations.

§ 1. For simplicity, let us consider a rectangular ocean basin of length L and of uniform depth H . Let the co-ordinate axes be taken as in the figure, the x -axis being taken in the mean free surface and the z -axis vertically downward.

Fig. I.



We shall denote by :

ρ the density of the water, a function of x and z ,

 μ the viscosity,

u, w the component velocities of current, in the directions of x and z respectively,

p the pressure at a point (x, z) ,

I Pub. Earthq. Inves. Com., **18**, 26 (1904);
Bull. Imp. Earthq. Inves. Com., **2**, 47 (1908).

2 Tokyo Sugaku-Buturigakkwai Kizi., 4, 384 (1908).

p_0 the external pressure over the free surface, which is assumed to be constant,

ζ the depression of the free surface from the mean level,

t the time,

g the acceleration due to gravity.

Now the fundamental equations of motion of a viscous liquid in two dimensions are :

$$\frac{Du}{Dt} = X - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \Delta^2 u \dots \dots \dots (1),$$

$$\frac{Dw}{Dt} = Z - \frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{\mu}{\rho} \Delta^2 w \dots \dots \dots (2),$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + w \frac{\partial}{\partial z},$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}.$$

Since our ocean has a vast extent compared with its depth, as if it were merely a thin lamina, the quantities w and $\frac{\partial u}{\partial x}$ may be neglected compared with its horizontal motion, unless there were a very abrupt local disturbance. In our case we have $X=0$, $Z=g$. Hence, the above equations reduce to :

$$\frac{\partial u}{\partial t} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial z^2} - \frac{1}{\rho} \frac{\partial p}{\partial x} \dots \dots \dots (1_a)$$

$$p - p_0 = \int_0^z g \rho \cdot dz \dots \dots \dots (2_a)$$

viz., the pressure at any point (x, z) is sensibly equal to the statical pressure due to the depth below the surface.

Further let us assume that in the upper strata of the sea of the thickness H_1 the density of water ρ differs from place to place horizontally but not vertically, and in the remaining lower strata of the thickness H_2 , the water is homogeneous throughout, and also that the difference of the density at any two points is so small that its higher orders can be neglected. In these circumstances, the equation (2_a) gives

$$\begin{aligned} -\frac{\partial p}{\partial x} &= g \left(\rho \frac{\partial \zeta}{\partial x} - \frac{\partial \rho}{\partial x} z \right) \\ &= b(D-z) \quad \text{for the upper strata } 0 < z < H_1 \\ &= b(D-H_1) \quad \text{for the lower strata } H_1 < z < H, \end{aligned}$$

where

$$\left. \begin{aligned} b &= g \cdot \frac{\partial \rho}{\partial x} \text{ (known),} \\ D &= \rho \cdot \frac{\partial \zeta}{\partial x} \text{ (unknown)} \end{aligned} \right\} \dots\dots\dots (3).$$

It is here to be noticed that, in our assumed ocean, both of these quantities, b and D , must be independent of z , but may depend on x and t only, and also that D corresponds to the depth of the Grenzfläche, because $\frac{\partial \rho}{\partial x} = 0$ when $z = D$.

Thus, if we confine ourselves to the final state in which the flow has become steady, and if by u_1 and u_2 we denote the horizontal currents in the upper heterogeneous and lower homogeneous layers respectively, the dynamical equation (1_a) takes the form

$$\left. \begin{aligned} \frac{\partial^2 u_1}{\partial z^2} + \frac{b}{\mu} (D - z) &= 0 \text{ for } 0 < z < H_1 \\ \frac{\partial^2 u_2}{\partial z^2} + \frac{b}{\mu} (D - H_1) &= 0 \text{ for } H_1 < z < H \end{aligned} \right\} \dots\dots\dots (4),$$

and the equation of continuity may be written

$$0 = \int_0^H u \cdot dz = \int_0^{H_1} u_1 \cdot dz + \int_{H_1}^H u_2 \cdot dz \dots\dots\dots (5).$$

As the boundary conditions, we shall take :

At the free surface

$$\left. \frac{\partial u_1}{\partial z} \right|_{z=0} = 0 \dots\dots\dots (6).$$

At the intermediate surface of discontinuity $z = H_1$

$$\left. u_1 = u_2 \right|_{z=H_1} \dots\dots\dots (7),$$

$$\text{and } \left. \frac{\partial u_1}{\partial z} = \frac{\partial u_2}{\partial z} \right|_{z=H_1} \dots\dots\dots (8).$$

At the bottom, according to the assumption,

$$\text{either } \left. u_2 \right|_{z=H} = 0 \dots\dots\dots (9a),$$

$$\text{or } \left. \frac{\partial u_2}{\partial z} \right|_{z=H} = 0 \dots\dots\dots (9b).$$

At the end-walls

$$u = 0 \text{ for } x = 0 \text{ and } x = L \dots\dots\dots (10).$$

II. Solutions.

§ 2. Case where the bottom condition is to be taken $|u_2|_{z=H}=0$,
i. e., no bottom-current.

For the upper strata, the first integral of equation (4) is

$$\frac{\partial u_1}{\partial z} = \frac{b}{2\mu}(z^2 - 2Dz) \dots\dots\dots (11),$$

the integration constant being 0 by surface condition (6).

Integrating again there results

$$u_1 = \frac{b}{6\mu}(z^3 - 3Dz^2 + B_1) \dots\dots\dots (12).$$

And for the lower strata, the first integral is

$$\frac{\partial u_2}{\partial z} = \frac{b}{2\mu}\{2(H_1 - D)z - H_1^2\} \dots\dots\dots (13),$$

the integration constant being taken to satisfy condition (8).

The second integral is therefore

$$u_2 = \frac{b}{2\mu}\{(H_1 - D)z^2 - H_1^2 z + B_2\} \dots\dots\dots (14),$$

and this, by the bottom condition (9a), becomes

$$u_2 = -\frac{b}{2\mu}\{(H_1 - D)(H^2 - z^2) - H_1^2(H - z)\} \dots\dots\dots (14a)$$

Now the constant B_1 in equation (12) can be determined by condition (7), thus

$$B_1 = -\{3(H - H_1)(HH_1 - HD - H_1D) + H_1^3 - 3DH_1^2\}.$$

Hence, (12) finally reduces to

$$u_1 = \frac{b}{6\mu}\{(z^3 - 3Dz^2) - (H_1^3 - 3DH_1^2) - 3(H - H_1)(HH_1 - HD - H_1D)\} \dots\dots\dots (12a)$$

In order to test the equation of continuity, we first calculate the integrals $\int_0^{H_1} u_1 dz$ and $\int_{H_1}^H u_2 dz$.

$$\frac{2\mu}{b} \int_0^{H_1} u_1 dz = -\frac{H_1^3}{12}(3H_1 - 8D) - H_1(H - H_1)(HH_1 - HD - H_1D),$$

$$\frac{2\mu}{b} \int_{H_1}^H u_2 dz = -\frac{1}{3}(H_1 - D)(2H^3 - 3H^2H_1 + H_1^3) + \frac{1}{2}H_1^2(H - H_1)^2.$$

Hence if we put

$$m \equiv \frac{D}{H_1}, \quad n \equiv \frac{H}{H_1} \dots \dots \dots (15),$$

the equation of continuity gives

$$\frac{1}{12} \left[(12n^2 - 4)m - (12n^2 - 12n + 3) \right] = \frac{1}{6} \left[4n^3 - 9n^2 + 6n - 1 - 2m(2n^3 - 3n^2 + 1) \right],$$

$$\text{or,} \quad m \equiv \frac{D}{H_1} = \frac{8n^3 - 6n^2 + 1}{8n^3} \dots \dots \dots (16_a).$$

This determines the position of the Grenzfläche, which is now known to be a function of H and H_1 only, but not of x .

Next, it is required to find the position of the no-current layer.

First, in order that the no-current layer should exist just on the discontinuous surface ($z=H_1$), we must have

$$u_1 = u_2 = 0 \quad \text{at } z = H_1,$$

$$\text{i. e.,} \quad 0 = (H_1 - D)(H^2 - H_1^2) - H_1^2(H - H_1)$$

$$\text{or} \quad m = \frac{n}{n+1},$$

which, combined with (16_a), gives

$$\left. \begin{aligned} n &= H/H_1 = 2.752, \\ m &= D/H_1 = 0.735 \end{aligned} \right\} \dots \dots \dots (17_a).$$

If the given basin has $n < 2.752$, the no-current layer must lie in the upper part of heterogeneous density, and its depth can be determined by $u_1 = 0$, namely

$$\left(\frac{z}{H_1} \right)^3 - 3m \left(\frac{z}{H_1} \right)^2 = (1 - 3m) + 3(n-1)\{n - (n+1)m\}.$$

Cardan's solution of this cubic equation takes the form:

$$\frac{z}{H_1} = m + \left\{ \frac{K_a}{2} + m^3 + \sqrt{\frac{K_a^2}{4} + K_a m^3} \right\}^{\frac{1}{3}} + \left\{ \frac{K_a}{2} + m^3 - \sqrt{\frac{K_a^2}{4} + K_a m^3} \right\}^{\frac{1}{3}} \dots \dots \dots (18_a),$$

where

$$K_a = (1 - 3m) + 3(n-1)\{n - (n+1)m\}.$$

On the contrary, if $n > 2.752$, the no-current layer will lie in the lower part of homogeneous density and its position can be found by

$$u_2 = 0 = (H_1 - D)(H^2 - z^2) - H_1^2(H - z),$$

$$\text{i. e.,} \quad \frac{z}{H_1} = \frac{1}{1-m} - n = \frac{2n^3 + n}{6n^2 - 1} \dots \dots \dots (19_a).$$

§ 2_b. Case where the bottom condition is to be taken $\left| \frac{\partial u_2}{\partial z} \right|_{z=H} = 0$, i. e., no bottom-friction.

Equations (11) to (14) hold good, as they stand, in the present case too. But the bottom condition (9_b) is now

$$\left| \frac{du_2}{dz} \right|_{z=H} = \frac{b}{2\mu} \{ 2(H_1 - D)H - H_1^2 \} = 0,$$

which immediately gives the depth of the Grenzfläche

$$m \equiv D/H_1 = 1 - 1/2n \dots\dots\dots (16_b).$$

The remaining condition (7) and the equation of continuity (5) determine the two integration constants B_1 and B_2 . That is, from the condition that $u_1 = u_2$ at $z = H_1$, we have

$$H_1^3 + B_1 = 3B_2,$$

and by the equation of continuity

$$B_1 H_1 + \frac{H_1^3}{4} (H_1 - 4D) = - (H_1 - D) (H^3 - H_1^3) + \frac{3}{2} H_1^2 (H^2 - H_1^2) - 3B_2 (H - H_1).$$

Hence, putting $m \equiv D/H_1$, $n \equiv H/H_1$ as before, and having the relation (16_b) in mind, we obtain

$$B_1 = H_1^3 \cdot \frac{4n^2 - 4n + 1}{4n}, \quad \text{and} \quad B_2 = H_1^3 \cdot \frac{4n^2 + 1}{12n}$$

and so finally

$$u_1 = \frac{b}{6\mu} \left[z^3 - 3DZ^2 + \frac{4n^2 - 4n + 1}{4n} \cdot H_1^3 \right] \dots\dots\dots (12_b),$$

$$u_2 = \frac{b}{2\mu} \left[(H_1 - D)z^2 - H_1^2 z + \frac{4n^2 + 1}{12n} \cdot H_1^3 \right] \dots\dots\dots (14_b).$$

As to the no-current layer, it lies just at the discontinuous surface ($z = H_1$), if

$$|u_1|_{z=H_1} = 0 = H_1^3 - 3DH_1^2 + \frac{4n^2 - 4n + 1}{4n} \cdot H_1^3,$$

$$\text{viz.,} \quad \left. \begin{array}{l} n = 2.207, \\ m = 0.774 \end{array} \right\} \dots\dots\dots (17_b).$$

But if the given basin is of $n < 2.207$, the no-current layer will be at a depth such that

$$u_1=0=\left(\frac{z}{H_1}\right)^3-3m\left(\frac{z}{H_1}\right)^2+\frac{4n^2-4n+1}{4n},$$

which gives

$$\begin{aligned} \frac{z}{H_1}=m+\left\{m^3-\frac{K_b}{2}+\sqrt{\left(\frac{K_b}{2}\right)^2-K_b\cdot m^3}\right\}^{\frac{1}{3}} \\ +\left\{m^3-\frac{K_b}{2}-\sqrt{\left(\frac{K_b}{2}\right)^2-m^3\cdot K_b}\right\}^{\frac{1}{3}} \dots\dots\dots (18_b), \end{aligned}$$

where

$$K_b=\frac{4n^2-4n+1}{4n}.$$

And if $n>2.207$, the depth of the no-current layer is given by

$$u_2=0=(1-m)\left(\frac{z}{H_1}\right)^2-\frac{z}{H_1}+\frac{4n^2+1}{12n},$$

viz.,

$$\frac{z}{H_1}=n-\frac{1}{6}\sqrt{12n^2-6} \dots\dots\dots (19_b).$$

§ 3. In the preceding paragraphs we have treated two cases: in the one case we assumed the bottom-friction to be nil, and in the other the bottom-friction is taken to be great enough to keep the bottom-velocity always at zero. For the real sea, of course, the bottom condition will be between these two, and consequently all things in it may also be presumed to be between those in the above extreme cases.

III. Discussions

§ 4. *The difference of surface-level due to a difference in the density of water.*

Let $\bar{\zeta}$ =the supposed depression of the free surface from the mean sea-level if the water were in statical equilibrium,

ζ =the real deviation of level when dynamical equilibrium has been attained.

Let the distribution of density in the upper strata be given by

$$\rho=f(x)=\rho_0+\sum_1^{\infty}A_r\cos\frac{2\pi r}{L}x,$$

ρ_0 being the mean density at the place where $\zeta=0$.

Then, in statical equilibrium, we have

$$(H_1-\bar{\zeta})\cdot\rho=H_1\rho_0,$$

which gives

$$\bar{\xi} = \frac{\rho - \rho_0}{\rho} \cdot H_1 \div H_1 \left[\sum_1^{\infty} A_r \cos \frac{2\pi r}{L} \cdot x \right],$$

since ρ does not much differ from 1.

Similarly, in dynamical equilibrium, the weight of the column of water above the Grenzfläche being constant, we have

$$\xi = \frac{\rho - \rho_0}{\rho} \cdot D = D \left[\sum_1^{\infty} A_r \cdot \cos \frac{2\pi r}{L} x \right].$$

Hence

$$\frac{\xi}{\bar{\xi}} = \frac{D}{H_1} = m \dots \dots \dots (20).$$

Thus, the ratio of the level difference in dynamical equilibrium to that in statical is equal to the depth of the Grenzfläche measured as a fraction of the total depth of the part of heterogeneous water.

§ 5. The depth of the Grenzfläche.

Using the formulae for the Grenzfläche (16_a) and (16_b), we have calculated out table I.

Table I

$n \equiv \frac{H}{H_1}$	$m \equiv D / H_1$	
	(a) No bottom-velocity	(b) No bottom-friction
1	0.375 (= 3/8)	0.500 (= 4/8)
2	0.641	0.750
3	0.755	0.833
4	0.815	0.875
5	0.851	0.900
6	0.876	0.917
7	0.893	0.929
8	0.907	0.938
9	0.917	0.944
10	0.925	0.950
20	0.963	0.975
∞	1.000	1.000

From the formulae or table I, we see:

(1). If $n=1$, i. e., the local inequality of density extends equally throughout the whole depth from surface to bottom, the Grenzfläche will lie at 3/8 to 4/8* of the total depth of the basin, according to the bottom

* This coincides with the statement of Krümmel, loc. cit.

conditions.

This means also that, in such a case, the level difference produced by a difference of density in dynamical equilibrium will be only $3/8$ to $4/8$ of the value calculated statically.

(2). m increases with n , and approaches to unity as its limiting value.

In other words, the shallower the upper part of the heterogeneous density in comparison with the total depth of the sea, the nearer the Grenzfläche approaches to, but necessarily higher than, the surface bounding the upper heterogeneous and the lower homogeneous parts of the sea-water.

(3). The thinner the part in which inequality of water exists, the nearer the level difference in dynamical equilibrium approaches to that calculated statically; and when the part of heterogeneous water is less than about $1/10$ of the whole depth as in the actual ocean, the sea-surface will assume practically the same level as if it were in statical equilibrium.

§ 6. The depth of the no-current layer.

By formulæ (17) to (19) with suffixes a and b , we have prepared table II.

Table II

$n \equiv \frac{H}{H_1}$	z/H_1	
	(a) No bottom-velocity	(b) No bottom-friction
1	0.422	0.500
2	0.764	0.920
2.207	—	1.000
2.752	1.000	—
3	1.076	1.318
4	1.389	1.726
5	1.711	2.142
6	2.037	2.559
7	2.365	2.975
8	2.695	3.400
9	3.025	3.823
10	3.356	4.240
20	6.678	8.470
∞	$\sqrt{1/3} \cdot n = \infty$	$n/3 = \infty$

Comparing this with table I, we observe that:

(1). The no-current layer will generally be lower than the Grenzfläche.

The only exceptional case, in which the no-current layer coincides

with the Grenzfläche, is when the local inequality of density extends equally throughout the whole depth, and the bottom-friction may be taken as entirely zero.

(2). As $n \equiv H/H_1$ increases indefinitely, so also does the quantity z/H_1 ; and the no-current layer sinks down far below the depth H_1 .

§ 7. *The pressure at the bottom.*

Since on the Grenzfläche the pressure is everywhere the same, so the pressure at the bottom must obviously vary according to locality.

Let Δp_H denote the excess of bottom-pressure at a place over that at the place of mean density ρ_0 , then

$$\Delta p_H = g\rho(H_1 - D) - g\rho_0(H_1 - D) = g(\rho - \rho_0)(H_1 - D).$$

But we have already seen that the depression of the sea-level, ζ , is given by

$$\zeta = -\frac{\rho - \rho_0}{\rho} \cdot D$$

Introducing this into the above equation, we get

$$\Delta p_H = \frac{H_1 - D}{D} \cdot g\rho\zeta = \frac{1-m}{m} \cdot g\rho\zeta \dots \dots \dots (21).$$

Thus, the pressure at the sea-bottom is greater where or when the sea-level is lower, the excess Δp_H being $\frac{1-m}{m}$ of the weight of a water-column whose height is equal to ζ .

A table for $(1-m)/m$ is given below :

n	(1-m)/m	
	(a) No bottom-velocity	(b) No bottom-friction
1	1.667(=3/5)	1.000
2	0.560	0.333
3	0.325	0.205
4	0.227	0.143
5	0.175	0.111
6	0.141	0.091
7	0.120	0.076
8	0.107	0.066
9	0.090	0.059
10	0.081	0.053
20	0.038	0.026
∞	0	0

In the actual ocean whose n may be taken to be > 10 , the difference of the bottom-pressure due to the difference in the density of water will

be negligibly small, in accordance with Nagaoka's opinion.¹ But here we have two special cases worthy of notice.

The one is such that, as off the north-eastern coast of Japan, there warm and cold currents run side by side, and the local difference of density extends to the greater parts of the total depth of sea-bottom. In such a case, the part of the level difference, which is produced by the difference in density in the limited region, will affect the pressure at the sea-bottom, but in the sense opposite to Omori's assumption², i. e., the pressure at the bottom being rather small when the surface level is high.

The second case to be noticed is the shallow sea which covers the continental shelf fringing the deep ocean. Let h be the depth of the fringing sea, and suppose $h \ll D$, D being the depth of the Grenzfläche of the main ocean. Then, the bottom-pressure of the shallow sea will exceed the pressure at the same depth in the ocean of mean density ρ_0 by an amount:

$$\begin{aligned} \Delta p_h &= g\rho_0(D-h) - g\rho(D-h) = g(\rho_0 - \rho)(D-h) \\ &= -\frac{D-h}{D} \cdot g\rho\zeta \dots\dots\dots(22), \end{aligned}$$

ζ being the depression of the mean sea-level.

In a shallow sea, therefore, the pressure at the bottom will change directly proportional to the variation of the mean sea-level. Especially in a very shallow sea, only 10 or 20 meters deep, say, the rise or fall of the mean sea-level corresponds to the increase or decrease of the bottom-pressure nearly equal to the weight of a water-column of height $|\zeta|$. This is the only case in which Omori's assumption² on the change of pressure at the sea-bottom may be accepted.

1. Loc. cit.
2. Loc. cit.

The Causes of the Annual Variation of the Mean Sea Level along the Japanese Coast.

by

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(Received May 18, 1926)

ABSTRACT.

Which is the most effective and what is the relative importance, of the various causes of the annual variation of mean sea level, and how are they to be determined quantitatively? These questions are very longstanding, but have not yet been definitely solved. The authors have studied this problem in connection with the Japanese coast, and have been able to solve it quantitatively. According to their results, along the Japanese coast at least, the seasonal fluctuation of the mean sea level is chiefly produced by the variation of the sea water temperature. The variation of barometric pressure is next in importance, but not much stress need be put on the wind and all other causes.

Introduction.

It is a phenomenon common to all parts of the world that the mean sea level changes with the seasons of a year; in some parts¹ its annual inequality reaches even one or two meters. Along the Japanese coast, the mean sea level generally has its maximum elevation in August or September, and its minimum height in January or February, the range of the inequality being 20 or 30 cms.

Though the irregularity of the sun's motion may cause the semi-annual and the annual tides, those tides theoretically will be very feeble and can not exceed about 4% of the lunar semi-diurnal tide; so the causes of the annual variation of considerable amplitude as stated above must be

¹ R. Harris, *Manual of Tides*, IV, 456—465(1907)

sought elsewhere. Now it is the generally accepted view that the annual change is chiefly what is called the "meteorological tide", which is caused by changes of the atmospheric pressure, wind, temperature of the sea water, precipitation, evaporation, oceanic current, etc. Hitherto, however, there no method has been devised to analyse and calculate quantitatively the effect of those elements, so opinion is divided as to which one of these causes is the most effective and what is the relative importance of each cause.

Of course, the relative importance of these causes must be different according to the local conditions. As regards shallow inland seas or long narrow bays, E. Piccard¹ found that the annual variation of mean sea level at Kronstadt near the mouth of Finland Bay goes together with the change of precipitation in the neighbourhood. Ed. Brückner² noticed that the mean sea level at Odessa and of the Asow Sea in the Black Sea rises about 20 cms. in May or June on account of the increase in Spring of the quantity of water in the rivers in the southern parts of Russia. H. Lentz³ pointed out that the annual fluctuation of mean sea level along the German coast of the Baltic Sea is largely affected by the wind. According to O. Krümmel,⁴ the annual variation of sea level along the German, Danish, and Swedish coasts of the Baltic Sea ranges between 10 and 20 cms., and it is caused by precipitation, wind and bar metric pressure. Krümmel also states that the sea level along the coast of Aden Bay has about 25 cms. as its annual inequality and it may be explained as the influence of the monsoon. Notwithstanding that in all the said papers the investigators merely compared the variation of the sea level with that of the element believed to be its cause and found no more than that they run parallel to each other, it can be well understood that the sea level of shallow inland seas with narrow mouths or very narrow bays, as in the above examples, may be remarkably affected by the wind and tributary land water.

On the other hand, how will it be on the coast of the open sea in general? It is the effect of barometric pressure that was recognized universally from early times as an important cause of the seasonal fluctuation of sea level. According to Sir C. Close's review⁵ of the earlier investigations on this point, M. Daussy, already in 1831, studied the

1 Beitr. zur phys. Geogr. des Finn. Golfs, III(1903).

2 Meteor. Zeitschr., 297(1886).

3 Fluth und Ebbe, und die Wirkungen des Winds auf den Meeresspiegel, 120(1879).

4 Handbuch der Ozeanographie, I, 56(1907).

5 Geographical Journal, 52, 51 (1918).

effect of barometric pressure on sea level at Brest, and Sir J. Lubbock, Sir James Loss, W. Ferrel, and others followed him. In Japan, Dr. F. Omori¹ compared the level fluctuations at several tide-stations under our Military Survey Department with the variations of barometric pressure in the neighbourhoods.

But the real variations of the sea level can never be explained satisfactorily by the change of barometric pressure only. For instance, the effect of the barometric pressure at the tidal stations investigated by Dr. Omori was only 30 to 40 % of the actual level variation. Dr. Nagaoka² paid attention to the influence of the sea water temperature and said that it appeared to him that the fluctuation of sea water temperature would lead to annual variation of the sea level. After this suggestion, Omori³ again discussed the effect of water temperature. Both these investigations, however, are based on data of the temperature of the sea surface only, and not on the really observed data of subsurface temperature. Moreover, though it is obvious that the water must stand at a lower level where the density is great and at a higher in regions of low density, currents from higher level to lower are produced in order to diminish the level difference, as soon as an inclination of the level is generated. Thus the difference of level caused by the difference of density in open seas must generally be smaller than that in the case of statical equilibrium as in a communicating tube. Then, what ratio should they hold? Unless this point is made clear, no discussion can have much weight. For these reasons, the papers of Dr. Nagaoka and Dr. Omori are not decisive as to whether the water temperature variation is really the chief cause of annual fluctuation of mean sea level, but merely suggestive. Indeed, from early times it has been noticed by many learned men, Zöppritz, Mohn, etc., that the sea water temperature has some connection with the sea level, and it seems to be acknowledged that the water temperature variation should lead to some annual change of sea level. As for the importance of this influence, however, it appears to have been made light of. Krümmel⁴ said:

“Meere mit erheblichen Temperaturschwankungen vom Winter zum Sommer werden periodisch ihr Niveau heben und senken. Aber es handelt sich dabei nur um unbedeutende Wirkungen.”

1 Publications of the Earthquake Investigation Committee, **18**, 23(1904);

The Bulletin of the Imp. Earthquake Inves. Committee, **2**, 35(1908).

2 Proc. of the Tokyo Mathematico-physical Society, **4**, 382(1908).

3 Bulletin of the Imp. Earthquake Inves. Committee, **5**, 39(1911).

4 Handbuch der Ozeanographie, **I**, 35(1907).

R. Harris¹ insisted that the effect of water temperature variation must be so small that its annual inequality could not reach even 0.1 foot, and that the most important cause of annual variation of sea level throughout the earth would be the mechanical action of the wind. But his argument is too weak to be agreed with. He wrote:

"As a rule the water at most tidal stations stands highest in summer or autumn and lowest in the winter or spring. This suggests that the annual fluctuation may be due to the alternate heating and cooling of the ocean waters, causing alternate expansion and contraction of the volumes.....

"While it is undoubtedly true that the surface of the water in high latitudes stands on a slightly higher level than in low latitudes, and is higher in the early autumn than in the early spring, it can be easily seen that this annual fluctuation can not be considerable, i, e., it can scarcely be a measurable quantity. Suppose the surface of the water at a point in high latitudes to be 0.1 foot above the surface at a point 8000 miles away in the opposite hemisphere. The instantaneous slope will be 2 one-billionths. And so the accelerating force per unit mass will be 2*g* divided by one billion. But this acting through, say, 3 months will give rise to a velocity of 0.52 foot per second at the surface. If the assumed distance were less than 8000 miles, this velocity would be increased in proportion. As no such alternation of surface flow from one hemisphere to the other has been observed, it is practically certain that the results due to annual temperature changes in the water can not cause an annual inequality in sea level with a range as great as 0.1 of a foot and so this portion of it may be neglected."

In this estimate of Harris, the viscosity of sea water was neglected. It is, however, a well known fact that the kinetic eddy viscosity for a current of vast scale and incessantly disturbed by winds and waves, as in the ocean surface, is extraordinarily great; and so Harris' estimate is of little weight. Moreover, his insistence on the winds being the chief cause of sea level fluctuation was based only upon the fact that, in most places, the seasonal variations of the sea level are in harmony with the wind.

Reflecting upon the foregoing descriptions, we see that the former explanations as to the causes of the annual variation of mean sea level are almost all merely verbal or qualitative but not quantitative except that as to the effect of barometric pressure, and so naturally investigators keep to their own opinions and never endorse the views of others. Thus,

¹ *Manual of Tides*, IV, 455, 466 (1907).

in spite of this problem being very old, it may be said to be unsettled yet.

Now, in May of last year (1925), our Military Survey Department¹ published the results of determination of mean sea levels at the twelve tide-stations observed for the past twenty five years, from 1900 to 1924. The present authors were struck with the regular appearance nearly a simple harmonic, which the twenty five years mean of the annual variation of sea level assumes, and commenced to analyse it quantitatively and to determine the relative importance of its causes. The results are described below.

I. General Plan and Data of the Present Investigation.

Even if we except the effect of astronomical tide, the sea level is affected directly or indirectly by atmospheric pressure, wind, water temperature, precipitation, evaporation, oceanic current, sunshine, clouds, humidity, etc., that is, almost all meteorological elements. If we classify and arrange them suitably, we can reduce them to the following five as the causes of sea level fluctuation.

(I) Internal Causes :

- (i) Change in the quantity of the water itself due to precipitation, tributary land water, and evaporation.
- (ii) Density variation
 - (a) due to temperature variation caused by change of the sun radiation, wind, oceanic current, etc.
 - (b) due to salinity variation caused by the changes in precipitation, tributary land water, evaporation, oceanic current, etc.

(II) External Causes :

- (iii) Barometric pressure variation.
- (iv) Mechanical action of the wind.
- (v) Coriolis' force due to the earth's rotation, acting on the oceanic current.

'Internal Cause' means such as can be reduced to a change in water itself and 'External Cause' such as exists necessarily outside of the water.

In order to analyse quantitatively the annual variation of the mean sea level along the Japanese coast, the present authors calculate first the effects of barometric pressure and of water density, which are the most

important of the causes and can be calculated easily and determinately. Then, subtracting both these effects from the actual variation of sea level, they go on to discuss the residue as the influences of the wind and Coriolis' force. Though it is impossible to treat the latter influences in a general way, they can be estimated, as will be seen later, quantitatively, albeit roughly.

The influence of the change in the quantity of sea water itself, caused by precipitation, tributary land water, and evaporation of sea water, is the only one which we can not determine directly. But it may be supposed that this influence would perhaps be inappreciable in the case of small islands, such as our country, where no large river can be found and which are situated in a wide open sea. After subtracting the calculated fluctuations due to all those causes except the change in the quantity of water from the actual variation of sea level, we can see whether this presupposition is true or not.

As to the data used, the monthly means of the levels at 12 tide stations are supplied by the publication of the Military Survey Department¹ as already mentioned, and the corresponding barometric pressures at the nearest meteorological stations are taken from the reports of the Central Meteorological Observatory.² The data of water temperatures and salinities we owe to the Central Fishery Institute.³

The reports of the Institute contain abundant data of sea water temperature and salinity, but unfortunately the subsurface measurements in the seas adjacent to the tidal stations are few and irregular and to get complete sets of subsurface data throughout the year is much more difficult. Thus, of the 12 tidal stations, we can find only six having subsurface data with regard to the neighbouring seas for 4 or 5 years (from 1919 to 1922 or 1923) appropriate to our present object.

The 12 tidal stations, the 12 meteorological observatories, and the 6 lines of subsurface measurements are shown in the following table and map.

Finally, regarding the winds and currents on the sea surface, we consulted the various publications of the Japanese Hydrographic Department.⁴

1 The Results of Determination of Mean Sea Levels (1925).

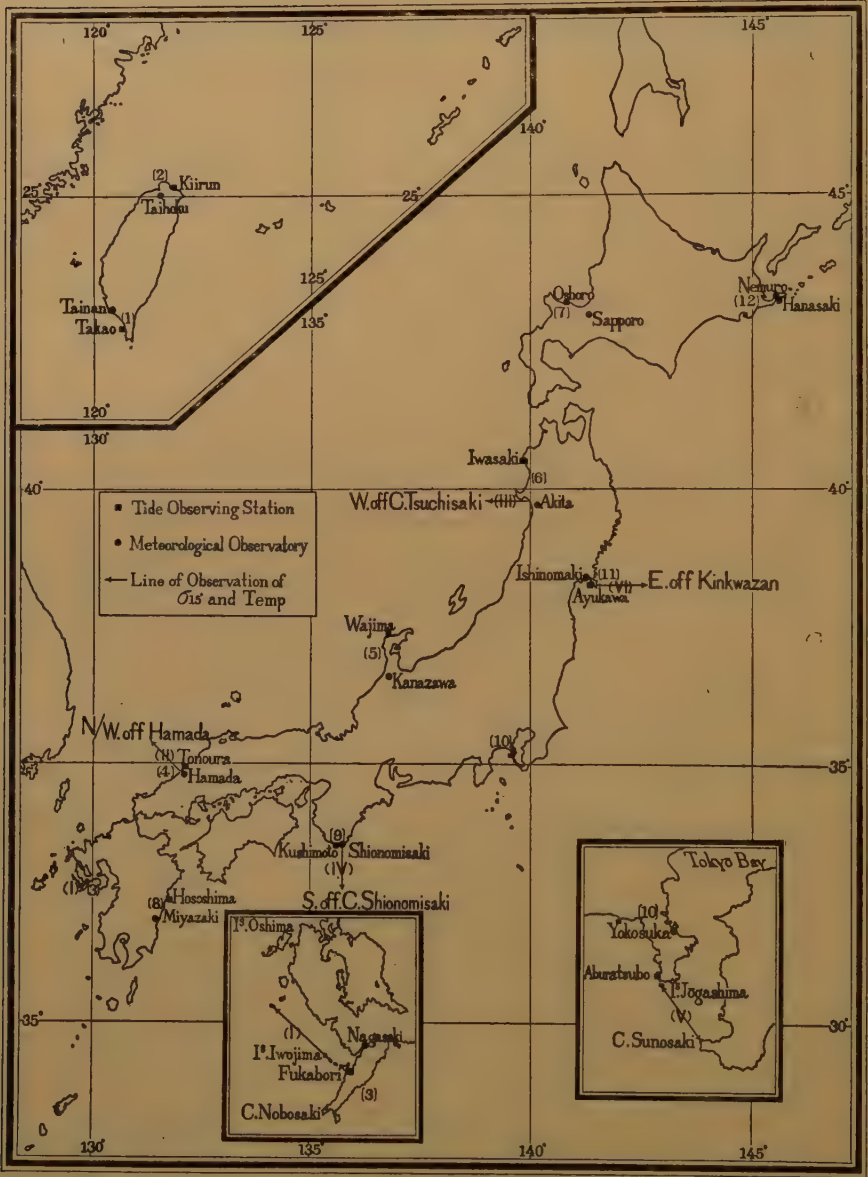
2 The Annual Reports (1914—1923).

3 The Reports of the Fundamental Fishery Investigations (1919—1923).

4 Meteorological Charts of the Sea adjacent to Japan (1918);

Current Charts of the North Pacific Ocean (1918);

Hydrographic Magazine, 2, 7, 36, 38 (1922, 1923, 1925, 1926).



Tidal Stations	Meteor. Stations	Lines of Subsurface Measurements
1. Takao	Tainan	
2. Kii-run	Taihoku	
3. Fukabori	Nagasaki	I. Iwôjima to I. Ôshima (i)
4. Tonoura	Hamada	N/W off Hamada (ii)
5. Wajima	Kanazawa	
6. Iwasaki	Akita	W. off C. Tsuchisaki (iii)
7. Oshoro	Sapporo	
8. Hososhima	Miyazaki	
9. Kushimoto	Shionomisaki	S. off C. Shionomisaki (iv)
10. Aburatsubo	Yokosuka	I. Jôgashima—C. Sunosaki (v)
11. Ayukawa	Ishinomaki	E. off Kinkwazan (vi)
12. Hanasaki	Nemuro	

II. The Effect of the Barometric Pressure.

§ 1. *The Pressure Factor.*

It is evidently true that the sea water stands higher with low barometric pressure and lower with a high barometer. If the sea water is in a state of statical equilibrium with regard to the atmospheric pressure, the equivalent change of the sea level corresponding to an atmospheric pressure change of 1 mm. Hg., that is, the so-called "pressure factor" should take the value :

$$\frac{\text{Specific gravity of mercury}}{\text{Specific gravity of sea water}} = 13.2.$$

Almost all the values of the pressure factor in the open sea given by former investigators approximate to the above value; but in some localities, the estimates range from 7 to 18. Though the factor may thus appear to be somewhat uncertain, it may be noted that places where the factor has a very small value are situated up a river or along the coast of an inland sea having a narrow mouth; on the other hand, the factor may perhaps be notably great if the proper vibration of a bay and

the change of the atmospheric pressure have nearly a coincident period. Hence the abnormal values of the pressure effect can be satisfactorily explained. Now, as we are going to deal with a large open sea where the water can move freely and with an annual fluctuation which runs very gradually, it can undoubtedly be taken that the actual pressure factor should be equal to the statical one mentioned above. Recently Mr. S. Ogura¹ investigated the pressure factor for the coast of small isolated-islands in the western part of the North Pacific Ocean. His results give a clear justification of the above remark.

For these reasons, we adopt the value 13.2 as the pressure factor in the following calculations.

§ 2. *The Calculations.*

We can assume the monthly mean of the sea levels as the so-called mean sea level.

At first we take the ten years' mean of the monthly means of sea levels at the twelve tidal stations (from 1914 to 1923). Next we take the corresponding ten years' mean of the monthly means of atmospheric pressure at the nearest meteorological observatories, and multiplying these by the pressure factor, calculate the effect of the atmospheric pressure on the mean sea level. Then we subtract the influence of the barometric pressure from the actual mean sea level.

The results of the calculations are as in table I and the annexed figures.

¹ Japanese Journal of Astronomy and Geophysics, 2, 1 (1925).

Table I.

[1] Takao (Tainan). 1914—1923.												
Month Variation	1	2	3	4	5	6	7	8	9	10	11	12 Range
Actual Sea Level (cm)	-13.0	-12.4	-6.3	-5.4	+0.6	+5.4	+10.8	+13.6	+12.1	+6.4	-1.4	-11.0 26.6
Baro. Press. (mm. Hg.)	764.8	762.7	762.1	759.7	757.2	755.7	754.9	753.6	756.0	759.6	761.8	763.2 11.2
Effect of Baro. Press. (cm)	-7.3	-4.5	-3.7	-0.5	+2.8	+4.8	+5.8	+7.6	+4.4	-0.4	-3.3	-5.2 14.9
Residual (cm)	-5.7	-7.9	-2.6	-4.9	-2.2	+0.6	+5.0	+5.9	+7.7	+6.8	+1.9	-5.8 15.6

[2] Kiirun (Taihoku). 1914—1923.												
Month Variation	1	2	3	4	5	6	7	8	9	10	11	12 Range
Actual Sea Level (cm)	-13.7	-12.7	-12.0	-6.0	+2.5	+9.6	+15.8	+17.7	+14.1	+3.3	-5.4	-12.9 31.4
Baro. Press. (mm. Hg.)	767.0	764.4	763.7	760.5	757.7	755.4	754.2	753.5	756.9	761.4	764.1	765.5 13.5
Effect of Baro. Press. (cm)	-8.8	-5.3	-4.4	-1.3	+3.6	+6.1	+8.2	+9.1	+4.6	-1.3	-4.9	-6.3 17.9
Residual (cm)	-4.9	-7.4	-7.6	-4.7	-1.1	+3.5	+7.6	+8.6	+9.5	+4.6	-0.5	-6.6 17.1

Table I (Continued).

[3] Fukabori (Nagasaki). 1914-1923.

Variation	Month	1	2	3	4	5	6	7	8	9	10	11	12	Range
Actual Sea Level (cm)		-17.0	-16.2	-14.4	-8.9	+0.9	+5.4	+13.2	+19.9	+18.0	+8.6	+1.4	-10.9	36.9
Baro. Press. (mm. Hg.)		767.0	764.9	764.3	761.7	758.7	756.2	756.7	755.5	758.8	762.7	765.7	766.5	111.5
Effect of Baro. Press. (cm)		-8.5	-4.4	-3.6	-0.1	+3.8	+7.2	+6.5	+8.1	+3.7	-1.5	-6.1	-6.5	16.6
Residual (cm)		-8.5	-11.8	-10.8	-8.8	-2.9	-1.8	+6.7	+11.8	+14.3	+10.1	+7.5	-4.4	25.6

[4] Tonoura (Hamada). 1914-1923.

Variation	Month	1	2	3	4	5	6	7	8	9	10	11	12	Range
Actual Sea Level (cm)		-14.4	-17.9	-17.7	-12.2	-3.5	+7.3	+15.7	+20.6	+15.9	+9.4	+3.0	-6.5	38.5
Baro. Press. (mm. Hg.)		765.5	764.1	763.8	761.3	758.3	755.9	756.4	755.9	759.4	762.8	764.9	765.2	9.6
Effect of Baro. Press. (cm)		-5.8	-4.0	-3.6	-0.3	+3.7	+6.9	+6.2	+6.9	+2.3	-2.3	-5.0	-5.4	12.7
Residual (cm)		-8.6	-13.9	-14.1	-11.9	-7.2	+0.4	+9.5	+13.6	+13.6	+11.7	+8.0	-1.1	27.7

Table I (Continued).

[5] Wajima (Kanazawa). 1914-1923.

Month Variation	1	2	3	4	5	6	7	8	9	10	11	12	Range
Actual Sea Level (cm)	-4.7	-11.9	-13.7	-13.2	-7.3	+1.7	+9.8	+15.3	+11.8	+6.5	+5.4	+0.1	29.0
Baro. Press. (mm. Hg.)	764.2	763.2	763.5	761.6	758.8	756.8	757.3	756.3	759.4	762.1	764.3	764.1	8.0
Effect of Baro. Press. (cm)	-4.3	-2.9	-3.3	-0.8	+2.9	+5.6	+4.9	+6.3	+2.1	-1.5	-4.4	-4.1	10.6
Residual (cm)	-0.4	-9.0	-10.4	-12.4	-10.2	-3.9	+4.9	+9.0	+9.7	+8.0	+9.8	+4.2	22.1

[6] Iwasaki (Akita). 1914-1923.

Month Variation	1	2	3	4	5	6	7	8	9	10	11	12	Range
Actual Sea Level (cm)	-5.2	-14.4	-14.1	-12.3	-5.8	+3.9	+10.0	+15.0	+11.0	+5.9	+5.3	+0.6	29.4
Baro. Press. (mm. Hg.)	761.6	762.4	762.4	761.4	758.8	756.7	757.3	756.7	759.9	762.6	763.8	762.1	7.1
Effect of Baro. Press. (cm)	-1.5	-2.5	-2.5	-1.2	+2.3	+5.0	+4.2	+5.0	+0.8	-2.8	-4.4	-2.1	9.4
Residual (cm)	-3.7	-11.9	-11.6	-11.1	-8.1	-1.1	+5.8	+10.0	+10.2	+8.7	+9.7	+2.7	22.1

Table I (Continued).

[7] Oshoro (Sapporo). 1914-1923.

Variation	Month	1	2	3	4	5	6	7	8	9	10	11	12	Range
Actual Sea Level (cm)		-3.9	-10.6	-11.6	-8.9	-2.2	+3.5	+8.6	+11.9	+8.7	+3.1	+1.9	-0.5	23.5
Baro. Press. (mm. Hg.)		759.7	761.2	760.7	759.9	758.0	756.2	756.9	756.9	760.1	762.2	762.1	759.7	6.0
Effect of Baro. Press. (cm)		-0.3	-2.3	-1.6	-0.5	+2.0	+4.4	+3.4	+3.4	-0.8	-3.6	-3.4	-0.3	8.0
Residual (cm)		-3.6	-8.3	-10.0	-8.4	-4.2	-0.9	+5.2	+8.5	+9.5	+6.7	+5.3	-0.2	19.5

[8] Hososhima (Miyazaki). 1914-1923.

Variation	Month	1	2	3	4	5	6	7	8	9	10	11	12	Range
Actual Sea Level (cm)		-14.5	-11.0	-12.1	-8.0	-0.9	+6.2	+8.1	+14.5	+14.1	+10.9	+1.7	-8.5	29.0
Baro. Press. (mm. Hg.)		766.3	764.2	763.9	761.6	758.7	756.6	757.1	755.8	758.7	762.4	765.3	765.8	10.5
Effect of Baro. Press. (cm)		-6.5	-3.7	-3.3	-0.3	+3.6	+6.4	+5.8	+7.4	+3.6	-1.3	-5.2	-5.8	13.9
Residual (cm)		-8.0	-7.3	-8.8	-7.7	-4.5	-0.2	+2.3	+7.1	+10.5	+12.2	+6.9	-2.7	21.0

Table I (Continued).

[9] Kushimoto (Shionomisaki). 1914-1923.

Variation	Month	1	2	3	4	5	6	7	8	9	10	11	12	Range
Actual Sea Level (cm)		-12.3	-10.2	-14.2	-8.1	-1.1	+5.8	+6.4	+13.6	+12.5	+10.9	+2.7	-5.7	27.8
Baro. Press. (mm. Hg.)		763.9	763.0	763.2	761.6	758.7	756.7	757.7	756.4	759.1	762.2	764.4	763.7	8.0
Effect of Baro. Press. (cm)		-4.0	-2.8	-3.0	-0.9	+2.9	+5.6	+4.3	+6.0	+2.4	-1.7	-4.7	-3.7	10.7
Residual (cm)		-8.3	-7.4	-11.2	-7.2	-4.0	+0.2	+2.1	+7.6	+10.1	+12.6	+7.4	-2.0	23.8

[10] Aburatsubo (Yokosuka) 1914-1923.

Variation	Month	1	2	3	4	5	6	7	8	9	10	11	12	Range
Actual Sea Level (cm)		-6.1	-6.2	-9.7	-8.1	-2.1	+2.1	+3.2	+8.7	+8.9	+7.4	+2.6	-0.1	18.6
Baro. Press. (mm. Hg.)		762.1	761.8	762.2	761.4	758.8	757.1	757.8	757.1	760.2	762.1	763.8	762.0	6.7
Effect of Baro. Press. (cm)		-2.1	-1.7	-2.3	-1.2	+2.3	+4.4	+3.6	+4.4	+0.4	-2.1	-4.4	-2.0	8.8
Residual (cm)		-4.0	-4.5	-7.4	-6.9	-4.4	-2.3	-0.4	+4.3	+8.5	+9.5	+7.0	+1.9	16.9

Table I (Continued).

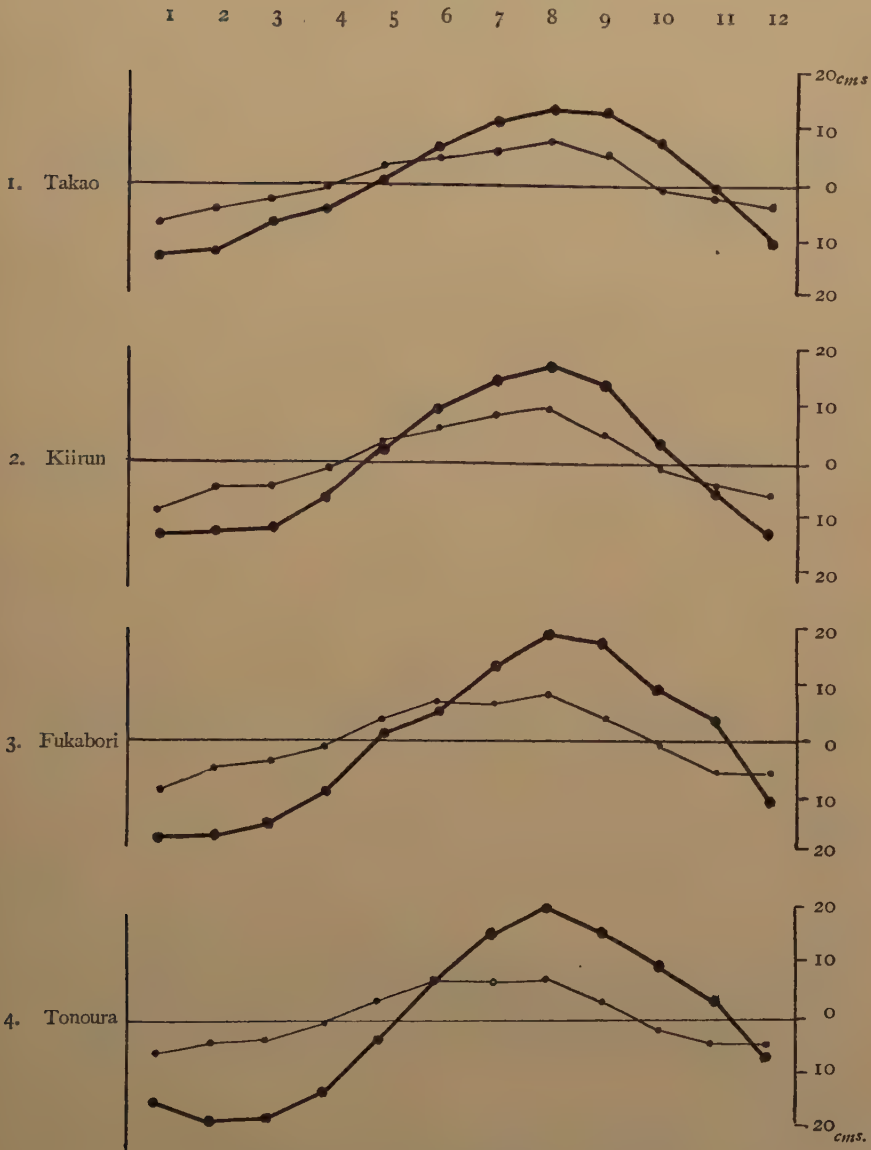
[11] Ayukawa (Ishinomaki). 1914-1923.

Month Variation	1	2	3	4	5	6	7	8	9	10	11	12	Range
Actual Sea Level (cm)	-3.4	-7.8	-11.9	-11.4	-6.1	-0.5	+4.4	+10.5	+10.6	+7.8	+4.8	+2.5	22.5
Baro. Press. (mm. Hg.)	761.8	761.7	762.4	761.6	759.2	757.2	757.9	757.4	760.4	762.9	762.8	761.9	5.7
Effect of Baro. Press. (cm)	-1.6	-1.5	-2.4	-1.3	+1.8	+4.5	+3.6	+4.3	+0.3	-3.1	-2.9	-1.7	7.6
Residual (cm)	-1.8	-6.3	-9.5	-10.1	-7.9	-5.0	+0.8	+6.2	+10.3	+10.9	+7.7	+4.2	21.0

[12] Hanasaki (Nemuro). 1914-1923.

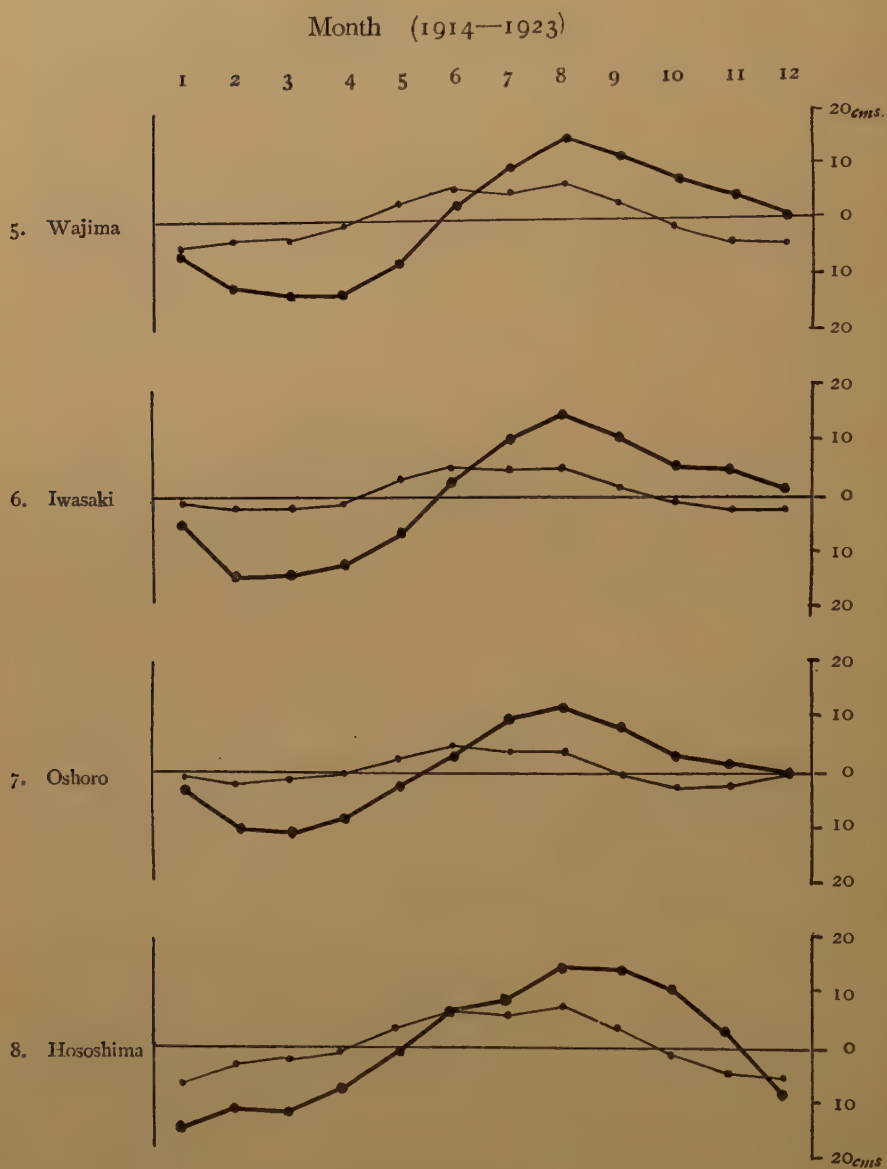
Month Variation	1	2	3	4	5	6	7	8	9	10	11	12	Range
Actual Sea Level (cm)	+6.9	-0.5	-3.4	-5.5	-4.6	-2.7	-0.6	+3.4	+2.0	-0.5	-1.1	+6.4	13.5
Baro. Press. (mm. Hg.)	757.9	760.5	760.0	760.1	759.0	757.4	758.3	758.0	760.8	762.2	762.2	758.2	4.8
Effect of Baro. Press. (cm)	+2.3	-1.2	-0.5	-0.8	+0.8	+2.9	+1.7	+2.1	-1.6	-3.4	-3.4	+1.9	6.3
Residual (cm)	+4.6	-0.7	-2.9	-4.8	-5.4	-5.6	-2.3	+1.3	+3.6	+2.9	+2.3	+4.5	10.2

Month (1914—1923)



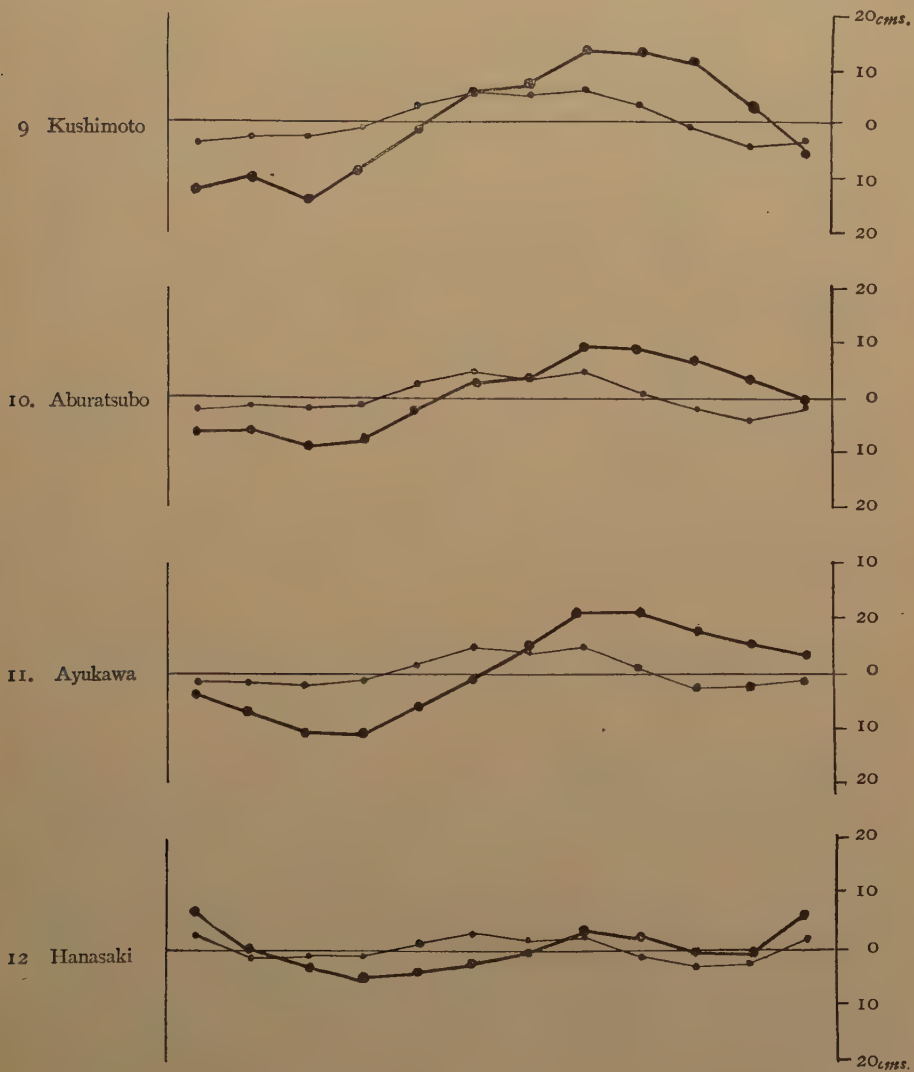
Thick lines.....Actual variations.

Fine lines.....Variations due to barom. pressure.



Month (1914—1923)

1 2 3 4 5 6 7 8 9 10 11 12



Thick lines.....Actual variations.

Fine lines.....Variations due to barom. pressure.

Let us here note the essential points in the above results.

1) The Range of Annual Variation due to Barometric Pressure.

The effect of the barometric pressure alone is not sufficient to account for even half the annual variation of sea level. This fact is numerically demonstrated by the following table.

Place		(I) Annual inequality of Actual M. S. L.	(II) Annual inequality of Effect of B. P.	(II) (I)
Tide Station	Met. Obs.			
1. Takao	Tainan	26.6 cms.	14.9 cms.	0.58
2. Kiirun	Taihoku	31.4 "	17.9 "	0.57
3. Fukabori	Nagasaki	36.9 "	16.6 "	0.45
4. Tonoura	Hamada	38.5 "	12.7 "	0.33
5. Wajima	Kanazawa	29.0 "	10.6 "	0.36
6. Iwasaki	Akita	29.4 "	8.0 "	0.27
7. Oshoro	Sapporo	23.5 "	8.0 "	0.34
8. Hososhima	Miyazaki	29.0 "	13.9 "	0.48
9. Kushimoto	Shionomisaki	27.8 "	10.7 "	0.39
10. Aburatsubo	Yokosuka	18.6 "	8.8 "	0.43
11. Ayukawa	Ishinomaki	22.5 "	7.6 "	0.34
12. Hanasaki	Nemuro	13.5 "	6.3 "	0.47

The ratio

$$\frac{\text{the annual inequality of the sea level due to barometric pressure}}{\text{the annual inequality of the actual mean sea level}}$$

takes the values of from 0.27 to 0.58.

2) The Phase of the Variation due to the Barometric Pressure.

The maximum effect of the barometric pressure appears in June, July, or August and the minimum in December or January. On the other hand, we see that the actual sea water stands at its highest level in August, and at its lowest level in January at Formosa, in February at Kyushu, and in March at the northern end of Japan.

Hence those two variations everywhere show a phase difference of one or two months, but we recognise only the roughly parallelism that both of them are at their maximum in summer and their minimum in winter.

3) The Residue after subtracting the Effect of Barometric Pressure from the Actual Mean Sea Level.

The residue after the barometric pressure effect is subtracted from the actual sea level variation is about $\frac{3}{4}$ of the total fluctuation.

The maximum of residual variation appears in September and its minimum in January at South Japan (Formosa), in February at Kyûshû, and is delayed as it goes to the north. Now, it is to be noted that the variation in the temperature of the water is quite parallel to this variation, and so the effect of change in density is perhaps remarkably greater.

III. The Effect of Change in Density.

§ 1. *The Effect of the Water Temperature and Salinity.*

The differences in the temperature and the salinity of the water can be reduced to the difference in density. As was stated in the introduction, in order to calculate the effect of the density variation, it is necessary to know the ratio between the difference in level caused by the difference in density in the open sea where the water can flow freely and that in the case of statical equilibrium.

One of the present authors¹ has discussed this question theoretically in his paper "on the so-called Grenzfläche in the current due to the difference of density." According to the result, in the case where the same inequality of density extends throughout the total sea water from surface to bottom, the real level difference should be only $3/8$ to $4/8$ of the statical one; in the other case, where the difference in density exists in the superficial layer only, for instance, in a layer of $1/10$ or so of the total depth, the actual level difference must be almost equal to the statical one.

Now, while the actual ocean on the earth has a mean depth of 4000 metres, local differences in the density of the water exist only in the upper stratum to a depth of 400 or 500 metres, below which the water is almost uniform throughout the whole ocean. The annual fluctuation of the water density, however, does not extend to such depths, commonly only to 200 or 300 metres. Hence, for the open sea coast, we may generally calculate the effect of density upon sea level variation as if in the case of statical equilibrium.

§ 2. *The Method of the Calculation.*

(i) We have data giving the water temperatures and salinities in the layers at depths of 0, 50, 100, and 150 or 200 metres in sea water.

From these data, we take possibly many years' means of the monthly mean temperatures and salinities, and find the densities corresponding to both the mean temperatures and the salinities with Knudsen's table.

¹ T. Nomitsu : These Memoirs, 10, III (1926).

It may be noticed that our data are the results of observations made only once in each month and that the dates of observations are different every year. We therefore take the mean of the really observed dates of the same month in the different years as the assumed date of observation in that month.

(ii) The inequality in density leads to a variation in specific volume. Let the specific volume be V_t and the density ρ_t , then

$$V_t = \frac{1}{\rho_t} = \frac{1}{1 + \frac{\sigma_t}{1000}}$$

where $\sigma_t = (\rho_t - 1) \times 1000$.

As the density inequality of sea water is of order of $1/1000$, we may put

$$V_t \approx 1 - \frac{\sigma_t}{1000}$$

Let two specific volumes V_t and V'_t correspond to σ_t and σ'_t respectively, then

$$\Delta V = V'_t - V_t = \frac{\sigma_t - \sigma'_t}{1000}$$

If we fix on a suitable density which is slightly greater than all of the obtained densities and find the differences between this standard and the other densities, we get the differences between the specific volumes corresponding to the standard and the other densities.

(iii) In order to get the total of the density effect, we must integrate the specific volume variation for the depth, namely ΔV , along the depth of the sea water. If we trace curve showing the relation between the range of the density variation (i. e. of the specific volume) and the depth of the water, we can determine the depth at which the variation becomes negligibly small. Here, using Simpson's rule, we integrate the value of ΔV from the surface to the depth where the density variation is approximately zero. The values obtained by the foregoing calculations are the heights referred to the sea level when the total sea water is assumed to have the standard density which we have fixed at first. Thus we can get the annual variation of sea level due to the change in the density of the water.

§ 3. Results of Calculations.

The results are summarised in table II and the annexed graphs.

Table II.

[3] Fukabori (Nagasaki). (i) I. Iôjima—I. Ôshima. 1919—1922.

Month	1	2	3	4	5	6	7	8	9	10	11	12	Range
Density(σ_t) Variation at Depth													
0m	25.39	25.93	25.72	25.59	24.81	24.02	22.46	21.71	22.05	23.28	23.90	24.72	4.22
50m	25.57	25.95	25.85	25.84	25.42	24.77	23.85	23.15	22.82	23.60	24.23	24.96	3.13
100m	25.67	26.03	25.88	26.00	25.74	25.51	25.15	24.87	24.77	24.87	24.96	25.05	1.26
150m	25.94	26.21	26.02	26.13	25.98	25.88	25.63	25.64	25.62	25.55	25.61	25.57	0.66
Effect of Density (cm)	-10.6	-15.7	-13.7	-13.4	-8.2	-1.5	+9.6	+16.8	+18.8	+12.4	+5.2	-1.7	34.5
Effect of Baro. Press. (cm)	-5.1	-4.1	-2.8	-0.4	+3.5	+6.3	+4.6	+6.6	+2.5	-1.5	-4.3	-5.8	11.7
Sum of Effects of Density and Baro. Press. (cm)	-15.7	-19.8	-16.5	-13.8	-4.7	+4.8	+14.2	+23.4	+21.3	+10.9	+0.9	-7.5	43.2
Actual Sea Level (cm)	-20.0	-14.5	-14.8	-12.1	-2.6	+7.6	+15.0	+20.1	+17.2	+11.8	+1.9	-9.2	40.1
Residual (cm)	-4.3	+5.3	+1.7	+1.7	+2.1	+2.8	+0.8	-3.3	-4.1	+0.9	+1.0	-1.7	

Table II. (Continued)

[6] Iwasaki (Akita). (iii) W. off C. Tsuchisaki. 1919-1923.

Month	1	2	3	4	5	6	7	8	9	10	11	12	Range
Density (σ_t) Variation at Depth	0m	25.91	26.46	26.51	25.69	24.31	23.68	22.35	22.12	23.15	23.84	24.98	4.39
	50m	26.07	26.59	26.61	26.18	25.87	25.76	24.43	24.33	24.48	24.21	25.18	2.40
	100m	26.09	26.62	26.56	26.25	26.47	26.33	25.36	25.45	25.48	25.27	25.36	1.35
	200m	26.37	26.73	26.87	26.56	26.46	26.77	26.25	26.40	26.49	26.52	26.45	0.62
Effect of Density (cm)	-8.4	-18.0	-16.5	-8.5	-6.5	-1.5	+0.5	+16.6	+13.7	+12.4	+11.7	+3.3	34.6
Effect of Baro. Press. (cm)	-1.8	-2.8	-2.8	-1.2	+2.4	+5.5	+2.8	+4.8	+0.9	-2.1	-2.8	-2.7	8.3
Sum of Effects of Density and Baro. Press. (cm)	-10.2	-20.8	-19.3	-9.7	-4.1	+4.0	+3.3	+21.4	+14.6	+10.3	+8.9	+0.6	42.2
Actual Sea Level (cm)	-6.4	-14.6	-15.0	-10.4	-5.3	+2.0	+10.4	+14.8	+9.6	+7.6	+7.4	+0.1	29.8
Residual (cm)	+3.8	+6.2	+4.3	-0.7	-1.2	-2.0	+7.1	-6.6	-5.0	-2.7	-1.5	-0.5	

Table II (Continued).

[9] Kushimoto (Shionomisaki). (iv) S. off C. Shionomisaki. 1919-1923.

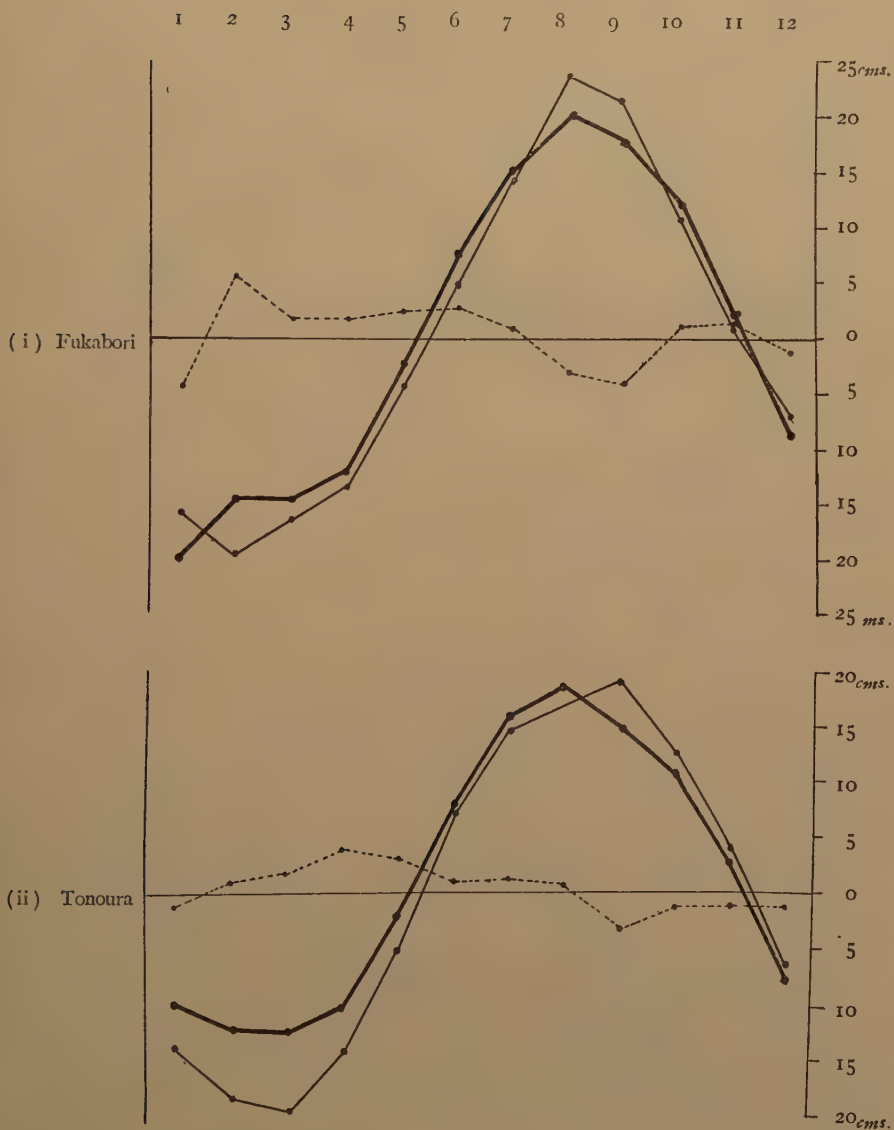
Month	1	2	3	4	5	6	7	8	9	10	11	12	Range
Density (σ_t) Variation at Depth	om	24.65	24.96	25.07	24.43	24.06	23.38	22.40	21.55	22.30	23.40	23.97	3.52
	50m	24.70	24.99	25.17	24.75	24.39	23.89	23.07	22.55	22.71	23.36	23.99	2.62
	100m	24.76	25.10	25.23	24.84	24.68	24.43	23.90	23.84	23.90	23.84	24.21	1.39
	200m	25.24	25.43	25.48	25.33	25.12	24.96	24.80	24.75	25.06	24.99	25.01	0.73
Effect of Density (cm)	-11.5	-17.2	-19.0	-12.6	-4.4		+4.9	+14.3	+18.3	+13.2	+10.3	+3.0	+1.5
Effect of Baro. Press. (cm)	-4.0	-2.8	-3.6	-1.0	+3.3	+6.4	+2.6	+2.6	+5.8	+2.4	-0.7	-3.6	-4.6
Sum of Effects of Density and Baro. Press. (cm)	-15.5	-20.0	-22.6	-13.6	-1.1	+11.3	+16.9	+16.9	+24.1	+15.6	+9.6	-0.6	-3.1
Actual Sea Level (cm)	-12.6	-10.9	-14.8	-6.7	+0.2	+8.0	+7.4	+7.4	+11.1	+12.2	+11.5	+0.8	-6.9
Residual (cm)	+2.9	+9.1	+7.8	+6.9	+1.3	-3.3	-9.5	-9.5	-13.0	-3.4	+1.9	+1.4	-3.8

Table II (Continued).

[10] Aburatsubo (Yokosuka). (v) I. Jōgashima—C. Sunosaki. 1919—1923.

Month	1	2	3	4	5	6	7	8	9	10	11	12	Range
Density (σ_t)	25.40	25.31	25.44	24.70	24.06	23.55	22.85	21.81	21.27	23.20	24.11	24.87	4.17
Variation	25.36	25.40	25.49	25.34	25.14	24.89	24.76	24.67	23.75	23.69	24.24	25.11	1.80
at	25.54	25.59	25.68	25.54	25.59	25.44	25.60	25.51	24.97	24.95	24.88	25.44	0.80
Depth	25.73	25.92	26.02	25.96	25.91	25.74	26.02	25.99	26.00	25.98	25.77	26.05	0.29
Effect of Density (cm)	-5.7	-7.3	-9.1	-7.5	-4.0	-0.3	+0.2	+3.5	+12.5	+12.3	+8.2	-0.3	21.5
Effect of Baro. Press. (cm)	-2.1	-2.0	-2.8	-1.5	+2.8	+4.6	+2.2	+4.2	+0.5	-1.5	-2.9	-2.0	7.5
Sum of Effects of Density and Baro. Press. (cm)	-7.8	-9.3	-11.9	-9.0	-1.2	+4.3	+2.4	+7.7	+13.0	+10.8	+5.3	-2.3	24.9
Actual Sea Level (cm)	-6.4	-6.7	-10.2	-7.9	-1.6	+4.3	+3.5	+5.3	+9.7	+9.0	+1.8	-1.3	19.9
Residual (cm)	+1.4	+2.6	+1.7	+1.1	-0.4	0	+1.1	-2.4	-3.3	-1.8	-3.5	+1.0	

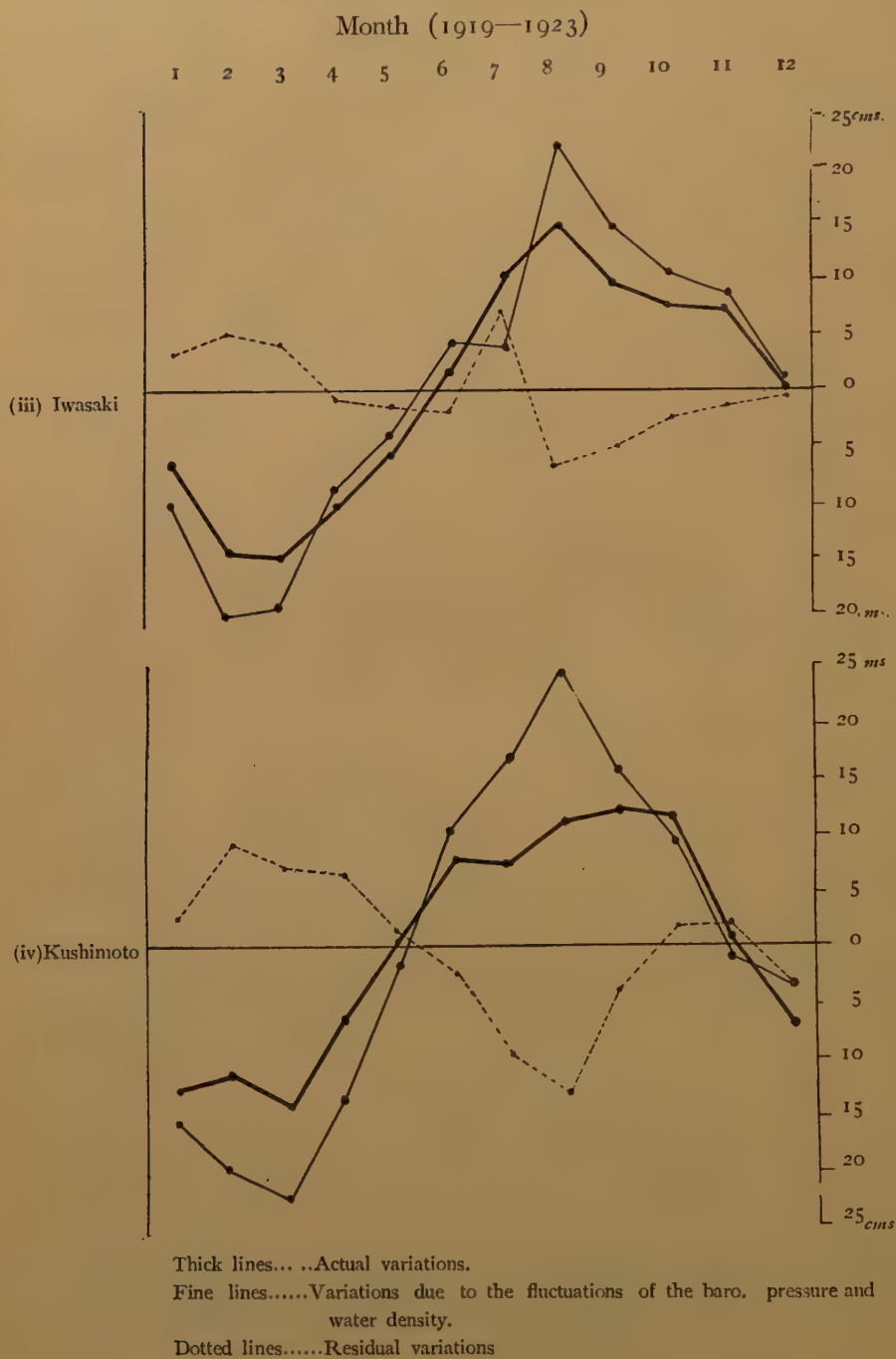
Month (1919—1922)



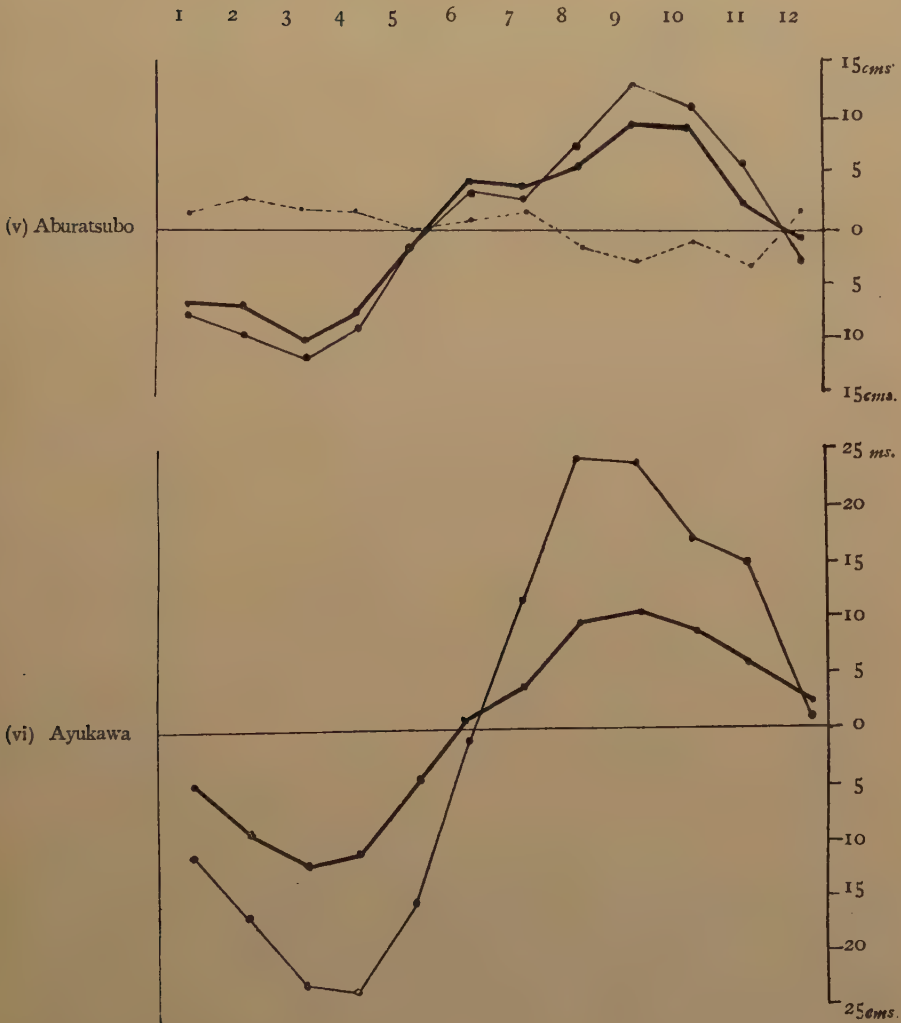
Thick lines.....Actual variations.

Fine lines.....Variations due to the fluctuations of the baro. pressure and water density.

Dotted lines.....Residual variations.



Month (1919—1923)



Thick lines.....Actual Variations.

Fine lines.....Variations due to the fluctuations of the baro. pressure and water density.

Dotted lines.....Residual variations.

Let us here note the essential point in the results of calculations.

1) The Range and Phase of Level Variation due to Change in Density.

First of all, the ranges of level variation are :

Place of Tidal Station : Observation of Temp. and σ_{15}°	(I) Range of the Actual Variation	(II) Range of Variation due to Density	(II) (I)
(i) Fukabori : Ōshima-Iwōjima	40.1 cms.	34.5 cms.	0.86
(ii) Tonoura : N/W off Hamada	36.1 "	31.3 "	0.89
(iii) Iwasaki : W. off Tsuchisaki	29.8 "	34.6 "	1.17
(iv) Kushimoto : S. off Shionomisaki	27.3 "	37.3 "	1.37
(v) Aburatsubo : Jōgashima-Sunosaki	19.9 "	21.5 "	1.08
(vi) Ayukawa : E. off Kinkwazan	22.4 "	45.5 "	2.05

From this table, we can see that the change in density accounts for most of the actual variation at the five positions except at Ayukawa.

The variation calculated as due to change in density at Ayukawa is very large as compared with the actual variation of mean sea level. Off Ayukawa, as a warm current (Kuroshio) and a cold current (Oyashio) intermingle each other, there is a conspicuous difference in density in a short distance and the difference extends from the surface to a very deep layer, almost to the bottom there. According to Nomitsu's theory¹ as to the "Grenzfläche", when the difference in density extends throughout a great part of the water depth, the actual difference in sea level caused by difference in density should be about half the statical one. This would explain the peculiarity at Ayukawa.

Next the maximum and minimum of sea level come out as follows :

Tidal Station	Maximum (cms)		Minimum (cms)	
	Actual Variation	Variation due to Density	Actual Variation	Variation due to Density
(i) Fukabori	+20.1 (VIII)	+18.8 (IX)	-20.2 (I)	-15.7 (II)
(ii) Tonoura	+18.5 (VIII)	+15.8 (IX)	-17.6 (III)	-15.5 (II)
(iii), Iwasaki	+14.5 (VIII)	+16.6 (VIII)	-15.0 (III)	-18.0 (II)
(iv) Kushimoto	+12.2 (IX)	+18.3 (VIII)	-14.8 (III)	-19.0 (III)
(v) Aburatsubo	+ 9.7 (IX)	+12.5 (IX)	-10.2 (II)	- 9.1 (II)
(vi) Ayukawa	+10.4 (IX)	+24.1 (IX)	-12.0 (III)	-20.8 (III)

1 Loc. cit.

Here I, II, III,.....mean January, February, April,.....respectively. Thus we see that the phase difference between the actual and the calculated sea level variations is very small.

2) The Relative Importance of the Effect of Salinity and of Water Temperature Change separately.

As the influence of temperature and that of salinity upon the density are not independent of each other, the effects of the two factors on sea level variation can not be completely separated and so we have determined both effects conjointly. But it is desirable to know their relative importance and therefore to separate them even approximately. For this purpose, we first assume the sea water at a certain place to have invariable salinity throughout the year equal to the yearly mean value of real salinity at that place, and calculate the effect of the temperature change alone. Subtracting this from the effect of the two factors conjointly, we get the effect of the salinity change alone. In order to show the relative importance of the temperature and the salinity change separately, we pick up from our calculations the amplitudes of sea level variation due to them.

Place	(I) Variation due to Density	(II) Variation due to Temp. only.	$\frac{(II)}{(I)}$	(III) Vari- ation due to Sal. only	$\frac{(III)}{(I)}$
(i) Fukabori	34.5 cms.	26.3 cms.	0.76	9.5 cms.	0.36
(ii) Tonoura	31.3 "	25.0 "	0.80	11.9 "	0.38
(iii) Iwasaki	34.6 "	34.6 "	1.00	8.9 "	0.26
(iv) Kushimoto	37.3 "	37.4 "	1.00	6.6 "	0.18
(v) Aburatsubo	21.5 "	20.2 "	0.94	4.5 "	0.21
(iv) Ayukawa	45.5 "	39.0 "	0.86	14.7 "	0.32
		mean	0.88	mean	0.29

3) The Effects of the Changes in Water Density and Barometric Pressure conjointly—Residue after subtracting them.

If we add the effects of density and barometric pressure calculated above, we see that, except in the case of Ayukawa, the result approaches very near the actual sea level in the whole run of variation throughout the year. Thus the annual variation of mean sea level can, for the most part, be taken as the effects of the change in density and of the variation in barometric pressure.

Precisely speaking, however, there still remains a residue, if we subtract the effects of barometric pressure and density change from the actual variation of mean sea level. The residues at Fukabori, Tonoura,

and Aburatsubo are very small and those at Iwasaki and Kushimoto are somewhat conspicuous.

But the sign of the residue is plus from winter to spring and minus from summer to autumn, and we see an obvious regularity in the mode of variation of the residue, for which some reasons will be sought in the following chapters.

IV. The Effect of the Wind.

§ 1. *The General Idea.*

It is generally accepted that the sea level is raised along the leeward and lowered along the windward coast. Of course, the quantitative effect of wind upon the sea level must differ very much according to the geographical features, the nature of the wind, and other circumstances, and there is perhaps no hope of getting a general law on this matter, the existing investigations of Colding¹ and Jeffreys² being only for very special cases.

Roughly speaking, however, we may classify the sea surfaces affected by the wind into the following two kinds.

(i) The first kind is a basin which is comparatively small in size and where the wind blows uniformly over the whole area. In this case, the sea water drifted on to the leeward coast can not run away laterally, but sinks down into the lower layers, and flows back horizontally on the bottom layer, in order to neutralize the level inclination.

Colding's experimental formula, based on observations in the Baltic Sea, can be used only in such cases.

Colding's formula states:

$$\zeta = 0.000000763 \frac{L}{H} W^2 \sin^2 \alpha,$$

where ζ is the difference of sea levels at the windward and the leeward coast (in metres),

L , the distance between the two coasts (in metres),

H , the mean depth (in metres),

W , the wind velocity (metres/sec.),

α , the angle between the wind and the coast line.

(ii) The second kind is an ocean of vast extent where the wind blows either over only small parts of the ocean surface so that the water can easily escape laterally along the leeward coast, or over the whole ocean but changing in direction from place to place so as to facilitate a complete circulation of the water on the sea surface.

¹ Kgl. Danske Vidensk. Selsk. Skrifter, 1, 272 (1880).

² Phil. Mag. 46, 114 (1923).

In such a case, the difference in level due to the wind will be comparatively small, because on the leeward side the sea water would run away before it accumulated to any great extent and on the windward side, water would be supplied from all around as soon as it was blown away by the wind. These considerations lead us to the view that the influence of wind on the annual variation of mean sea level must be rather small, which is contrary to Harris' opinion.

With regard to Japan, the coast of the Japan Sea wil belong to the first class, and the Pacific coast to the second class.

§ 2. *Estimation of the Effect of the Wind on the Japanese Coasts.*

Generally speaking, the prevailing wind over the seas adjacent to Japan is north westerly in winter and south easterly in summer.

Hence, on the Pacific coast, the effect of the wind ought to lower the level in winter and to raise it in summer. But we have seen that the reverse is the case, that is, the residue left after subtracting the effects of barometric pressure and of density inequality from the actual variation of the mean sea level is higher in winter and lower in summer. Hence, the effcet of the wind along the Pacific coast may perhaps be so feeble as to be unnoticeable compared with the effect of Coriolis' force.

On the contrary, on the Japan Sea side the residual variation obtained in the preceding chapter coincides with that required by the wind effect. Now noting that the prevailing wind over the Japan Sea is almost uniform throughout the whole surface, we will apply here Colding's formula.

According to the Meteorological Charts of the Seas adjacent to Japan published by our Admiralty and the Atlas of the Pacific Ocean by the Deutsche Seewarte, we may assume the wind velocity over the Japan Sea as below :

Seasons (Months)	Winter (I,II,III)	Spring (IV,V,VI)	Summer (VII,VIII,IX)	Autumn (X,XI,XII)
Wind velocity (vector mean)	6.2 m/sec. N. 20° W.	1.3 m/sec S. 40° E.	3.0 m/sec. S. 50° E.	4.8 m/sec. N. 15° E.

Assuming that $L=9 \times 10^5$ metres and $H=1530$ metres, we get the corresponding level variation as follows :

Seasons	Winter	Spring	Summer	Autumn
Level variation	+1.3 cms	-0.5 cm	-0.9 cm	+0.3 cm

V. The Effect of Coriolis' Force.

§ 1. *The Method of Estimation.*

Every moving body on the earth is affected by the earth's rotation and this effect is equivalent to the action of a deflecting force proportional to the velocity of the moving body. Let v denote the velocity of the moving body in latitude λ , then the deflecting force i. e., Coriolis' force per unit mass is

$$2v\omega\sin\lambda,$$

where ω represents the angular velocity of the earth's rotation (0.0000729 radian per second).

Now it is very difficult to determine what current would be generated and what influence would be affected upon the sea level by Coriolis' force, when the wind or other current-producing force is given. But if we have actually measured and know the state of the existing current, we can easily determine the effect of current upon sea level due to Coriolis' force. As for a free current which has escaped outside of the direct action of the current-producing force, the lateral equilibrium of the current gives the relation.

$$\text{Coriolis' force} - \text{Centrifugal force} = g \cdot \sin\gamma$$

where γ is the angle of level inclination across the current from right to left, and g is the gravity acceleration.

Hence, if the path, width, and velocity of the free current are known by actual measurements, we can calculate the inclination γ at any place. Then the total difference in level between the two sides of the current zone will be obtained as follows:

$$\text{The level diff.} = \Sigma \sin\gamma \Delta b,$$

where Δb is the elementary width of current zone having the velocity v and Σ means the sum of the values throughout the whole current zone.

In like manner, the sea level fluctuation due to the seasonal change of the oceanic current can be determined, provided that the current variations are known in detail.

§ 2. *Results of Estimation.*

Owing to the great difficulty of current measurement, even the various publications of the Hydrographic Department give us very insufficient data and so we can only very roughly estimate Coriolis' effect.

Along the side of Japan Sea, the Tsushima Current, a branch of the Kuroshio, flows north-east with a breadth of 30—50 sea-miles, and its average velocity determined by current-bottles is as below:

Season (Months)	Winter (I,II,III)	Spring (IV,V,VI)	Summer (VII,VIII,IX)	Autumn (X,XI,XII)
Current Velocity	8-9 mls/day	7-8 mls/day	8-10 mls/day	5-7 mls/day

This corresponds to a seasonal level variation of the following amount :

Season	Winter	Spring	Summer	Autumn
Level Variation	+1.2 cms.	-0.4 cms.	+1.6 cms.	-2.4 cms.

Along the Pacific coast runs the strong Kuroshio current and near the Kushimoto tidal station it has the following velocity and breadth.

Season	Winter	Spring	Summer	Autumn
Current Velocity	Left 20 mls wide Central 20 " wide Right 20 " wide	15 37 mls/day 20	? 38 mls/day ?	20 48 mls/day 22
Mean	24 mls/day	25 mls/day	30 mls/day	27 mls/day

The corresponding variation of sea level should be :

Season	Winter	Spring	Summer	Autumn
Sea Level variation	+5.0 cms.	+2.8 cms.	-7.1 cms.	-0.8 cms.

Lastly, let us compare these calculated effects of wind and Coriolis' force with those residues of sea level variation obtained in Chapter III.

On the Japan Sea side.

Season	Winter	Spring	Summer	Autumn
Residual variation	Tonoura Iwasaki	+0.6 cms. +4.6 "	+2.5 cms. -1.4 "	-0.8 cms. -1.5 "
mean	+2.6 cms.	+0.5 cms.	-1.2 cms.	-1.9 cms.
Coriolis' effect (calc.)	+1.2	-0.4	+1.6	-2.4
Wind effect (calc.)	+1.3	-0.5	-0.9	+0.3
Sum	+2.5	-0.9	+0.7	-2.1

On the Pacific Side.

Season	Winter	Spring	Summer	Autumn
Residual variation at Kushimoto	+6.6 cms.	+1.6 cms.	-8.6 cms.	-0.2 cm.
Current effect (calc.)	+5.0 cms.	+2.8 cms.	-7.1 cms.	-0.8 cm
Wind effect	?	?	?	?

Conclusions.

Reflecting on the results obtained in the foregoing chapters, we arrive at the following conclusions.

(i) Of the various causes of annual variation of mean sea level, the most effective is the influence of water density change due to inequalities in its temperature and salinity.

(ii) The effect of atmospheric pressure comes second in importance.

The above two effects conjointly approximate very near to the actual fluctuation of sea level.

(iii) The effect of the wind is commonly very slight, a conclusion which is in opposition to the opinion of Harris. On the Japan Sea side, the annual inequality due to wind effect reaches scarcely 5 cms., and along the coast on the Pacific side too it is far behind the effect of Coriolis' force.

(iv) The effect of Coriolis' force is also very small for the coast of the Japan Sea, but it is not so small for the Pacific coast where there is a strong oceanic current.

(v) The effect of change in the quantity of the water itself is negligible as presupposed.

It may be useful to remark here that, for the sea level fluctuation in a short period, say one or several days, the circumstances will be quite different. As the temperature and salinity of sea water remain nearly unchanged for several days, the short period variation of sea level due to the density change cannot be marked. On the other hand, the barometric pressure and the wind at most places fluctuate greatly from time to time, and therefore the sea level fluctuation in a short period must generally be governed by these two factors, especially by the barometric pressure. Mr. Ogura's investigation¹ upon the pressure factor may be taken to verify this point.

¹ Loc. cit.

Nevertheless, in a region where the variations of barometric pressure and the wind are very feeble, as in the equatorial calm zone, even a small effect of water density variation can not be neglected compared with the effect of barometric pressure and wind. The abnormally great value of the pressure factor obtained by Ogura for the equatorial region is perhaps due to his neglect of the effect of density.

Studies on Proteins, II. Action of Superheated Water on Proteins, I.

By

Shigeru Komatsu and Chuichi Okinaka.

(Received June 18, 1926.)

Taylor¹ has observed that when pure sterile globulin from bovine serum was kept in distilled water at ordinary temperature for 18 months, it was hydrolysed to proteoses and partly polymerised into an insoluble form, and also the leucine may be recovered from a sterile suspension of casein in pure water after the lapse of a year or more. These facts indicate that the hydrolysis of proteins in neutral solution does occur at ordinary temperature, though the velocity of the autohydrolysis of proteins was very slow under these conditions. The influence of heat upon the autohydrolysis of proteins is of considerable interest, since it was a generally accepted view that the reaction of hydrolysis is accompanied by the evolution of heat and that the heat-coagulation of proteins was considered to consist in the polymerisation of the amphoteric protein molecule with the elimination of water according to the following equation:²



and accordingly the effect of applying heat to a protein solution must be to shift the equilibrium in the direction of polymerisation, as would follow from van't Hoff's law.

The study of the action of superheated water on proteins was first

¹ J. Biol. Chem., **1**, 345 (1906); On Fermentation, p. 223 (1907).

² Robertson: The Protein; p. 141, 173 (1909).

tried by Vöhler¹, and his report was followed by contributions from Subain,² Lubavin³ and others⁴ on the isolation of cleavage products of proteins.

The next result of these researches was (1) to establish that the protein body suffers fairly complete hydrolysis at the temperature of steam, and that the equilibrium between the protein complex and the amino acids, which were regarded as the products of its hydrolysis, was shifted to the amino acids side; (2) to show the presence of leucine, tyrosin, aspartic acid, and atmid-bodies of complex nature in the reaction products of the proteins.

The extensive researches by R. Neumeister and R. H. Chittenden and F. S. Meara have given us much additional information concerning the mode of cleavage of the protein molecule, though there are different opinions regarding the nature of the process which have revealed an important connection with the hydrolytic process brought about by the digestive enzymes and also by dilute acids. Many investigators mentioned above, have been content merely to examine the products, especially from the view point of physiology, to find some analogies between the action of superheated water and gastric or pancreatic digestion. Such a too close adhearance to analogies might lead investigators into serious error as was already pointed out by Chittenden and Meara, and cause them to fail to pay attention to the unravelling of the constitution of proteins. Furthermore, it was doubtful whether the atmid-bodies would result in a definite chemical composition, in spite of the intensity and duration of heating by which they were produced.

No body paid any attention to the information of the insoluble substance from proteins by the action of superheated water, nor offered any examination of its relation to the atmid-bodies, all merely dwelling on the consitution of the latter substances. The present research was to heat pro-

1 Lieb. Ann., **41**, 238 (1842).

2 Subain: Hoppe-Seyler: Medicin-chem. Untersuch. **480** (1871)

3 Lubavin: Ber. D. Chem. Ges., **10**, 2237 (1883).

4 Hammersten: Z. physiol. Chem., **7**, 227 (1883); Krukemberg: Sitzungsberichte der Jenaischen Gesellschaft für Medicin, etc., 1886; Ademont: C. R., **105**, 222 (1887); Crismer: Host in Jahres Berichte f. Thierchemie für 1891, 19; Koukol-Yasnopolsky: Pflüg. Arch. Physiol., **12**, 85. R. Neumeister: Z. Biolog., **26**, 57 (1890); **36**, 420 (1896); E. Salkowski: Ibid., **34**, 190 (1896); **37**, 404 (1899); Blum u. Vaubel: J. prak. Chem., **56**, 396 (1897); **57**, 365 (1898); R. Bauer: Z. Physiol. Chem., **35**, 342 (1902); H. S. Stendel: Ibid., **35**, 540 (1902); R. H. Chittenden and F. S. Meara: J. physiol., **15**, 501 (1896).

teins with water in the isoelectric point, therefore, undertaken by the authors, to find some chemical relation between the insoluble substance and the atmid-bodies, to propose chemical constitution of protein, if possible, and also to explain the mechanism of the decomposition of protein by superheated water.

The materials employed in the present experiment were three different proteins; edestin, gliadin from wheat flour, and casein from cow's milk, which were prepared by the authors following the directions proposed by T. B. Osborne and his co-workers.¹ The results of the analysis of the samples are shown in the following tables:

Table I.

	Edestin	Gliadin	Casein
C.....	51.30	53.78	53.39
H.	6.86	6.79	9.10
N.	18.63	17.65	15.55
Ash	0.7	0.5	0.25
H ₂ O.	6.43	4.1	3.67
Amide-N.	1.88	4.30	1.61
Monoamino-N.....	10.87	12.25	10.31
Diamino-N.....	5.9	1.1	3.5
Melanin-N.	0.1	0.1	0.2
PH ²	6.3	5-6 (mean 5.7)	4.6

One gr. of protein was introduced with twenty c. c. of distilled water into a glass stoppered bottle of seventy c. c. capacity, which was previously washed with steam in order to remove completely any alkaline substance which might be generated from the glass wall by the action of steam. The bottle was carefully stoppered, and heated in a thermostat which was kept at the constant temperature of 120°. The bottle was opened after heating for a certain number of hours, and the fluid which had generally a yellow colour, and gave a neutral reaction to litmus paper, was poured into a flask. The insoluble residue separated from the coloured solution, was washed with cold water and then dried to constant weight at 105°. The solution and washings combined together made up to 50 c. c., and the PH value of the solution was determined electrometrically of one part

¹ J. Amer. Chem. Soc., **25**, 323 (1903).

² PH of the pure protein was determined by the electrometric method of the solution which was prepared by shaking the protein with distilled water in a stoppered bottle for 24 hours. According to Michaelis, the PH for edestin and for casein is 5.6 and 4.7 respectively, and Eto (J. Biochem., **3**, 373 (1924),) has given 6.6 for that of gliadin.

of the solution. Another part of the solution was evaporated on a water bath to dryness and dried at 110° to constant weight and then analysed.

The elementary composition and the distribution of nitrogen of both the insoluble residue and the solution were determined in the usual way, and the results shown in the following table.

Table II.
Quantities of Soluble and Insoluble Parts.
Edestin

Temperature	110°			120°			
Heating (hrs)	3	6	9	1	2	4	20
Sample (gr)	0.920	0.920	0.920	0.950	1.093	1.166	0.926
Soluble, part. (gr)	0.134	0.212	0.310	0.065	0.136	0.263	0.588
" (%)	14.3	22.6	33.0	6.9	12.5	22.6	63.4
Insol. part. (gr)	0.899	0.868	0.683	—	—	—	—
" (%)	97.7	94.3	94.2	—	—	—	—

Gliadin and Casein (120°)

Heating (hrs)	2	6	17	2	6	15.5
Sample (gr)	0.900	0.974	0.967	0.975	0.988	1.003
Soluble part. (gr)	0.012	0.057	0.140	0.054	0.097	0.289
" (%)	1.3	5.9	14.0	5.5	9.9	28.8
Insol. part. (gr)	0.923	0.933	0.837	—	0.941	0.727
" (%)	102.6	95.8	86.5	—	96.2	92.4

Table III.
Distribution of Nitrogen.

	Casein (120°)			Gliadin (120°)			Edestin (110°)		
Heating (hrs).	2	6	17	2	6	15.5	3	6	9
Total N. (%)	—	17.0	15.0	16.3	14.4	16.2	16.2	18.7	17.8
Amide N. (%)	—	4.5	2.6	—	4.6	5.0	3.0	3.4	2.7
Diam. N. (%)	—	3.3	2.0	—	0.7	0.8	3.7	4.4	4.7
Total N. (%)	16.0	15.3	15.5	—	15.2	23.1	16.9	16.2	18.3
Amide N. (%)	—	1.3	2.1	—	4.9	6.9	1.6	1.3	1.4
Diam. N. (%)	—	3.0	5.1	—	0.8	1.5	5.4	4.9	5.3

Table IV.
Elementary Analysis of Insoluble Part
and P_H of the Solution

Heating (hrs).	Edestin (120°)			Gliadin (120°)			Casein (120°)		
	C	H	P _H	C	H	P _H	C	H	P _H
0	51.36	6.86	6.3	53.87	6.79	5.7	53.39	9.13	4.6
1	51.80	7.55	7.3						
6				52.27	7.11	6.8	54.29	7.34	4.7
20	52.46	7.20	6.9	52.19	6.85	6.4	54.27	7.17	5.0

The constitution of the hydrolysate of protein by the action of super-heated water can be divided according to their solubility in water, into two parts; the insoluble and soluble parts, and the latter composed of a substance of complex nature such as proteose, and of simple amino-acids, and the quantity of the soluble substances as was indicated in the table, was increased in proportion to the reaction time. The rate of dissolution of protein in water, being different for individual protein, is greatest in edestin, and gliadin and casein were ranged in that order.

The total weight of the hydrolysate as will be seen in the table, exceeded in all cases the employed amount of the sample, indicating the hydrolysis of protein molecule occurred in the digestion.

Although the insoluble part has the same appearance as the mother protein, they are different essentially in composition from each other, the insoluble part resulting from edestin is rich in the content of carbon and hydrogen than the mother protein, while that from gliadin is poor in carbon but rich in hydrogen, and the insoluble substance derived from casein is rich in carbon but poor in hydrogen compared with the mother protein.

Generally, the insoluble substance has no definite chemical composition varying with the external condition under which the protein was submitted to the chemical action; the substance derived from edestin shows gradual increase in carbon- and nitrogen-content with laps of the reaction time whilst that from casein and gliadin kept almost constant value.

These facts led the authors to the conviction that the chemical composition of the detached groups from protein should vary with each individual protein according to the amino-acids of which it was composed.¹

¹ G. Trier: *Chemie der Pflanzenstoffe*, 470 (1924); O. Cohnheim: *Chemie der Eiweisskörper* III. Auf. 279 (1911).

Moreover, the study of the nitrogen distribution of the insoluble and soluble parts by the usual methods, supports the above idea with regard to the composition of the hydrolytic products and also suggests an hypothesis with respect to the nature of the process in which protin undergoes hydrolysis.

When edestin was digested, the groups containing amide-nitrogen were removed mostly at the begining of the reaction from the protein molecule, whilst those containing diamino-nitrogen were detached gradually, and consequently the hydrolysis of the protein proceeded in a manner in which the distribution of nitrogen in the residue and solution will approach, in the progress of reaction, that in the original protein. Casein, however, shows quite different behavior toward superheated water since the atomic groups containing diamino-nitrogen were removed mostly from the protein molecule at the first period of the reaction, and gliadin with respect to its behavior stands between these two proteins.

It was a noteworthy as that the total amide nitrogen of the reaction products at the various stage of the reaction, as will be seen in the case of edestin heated with water at 110° , exceeded that of the protein, and on the contrary there shows a corresponding diminution in the total diamino-nitrogen of the reaction products when composed with the original sample. These facts indicate the transformation of the diamino-nitrogen into amide-nitrogen, and also the phenomenon was noticed markedly in the case of casein, but in gliadin the reaction take place in the least degree.

Such conversion of nitrogen compounds seems to be more probable when referred to the transformation of glyoxalin by the action of benzoyl chloride and alkali at 0° into bis-benzoyl-amino-ethylene,¹ and also the behavior of tryptophane toward chemicals.² Consequently the process involved in the appearance of the insoluble residue by the action of superheated water on protein was regarded as taking place in at least two ways.

First there is a formation of an insoluble substance resulting from the removal of the prominent parts from the protein complex, and secondarily, one part of other cleavage products soluble in water, forming simultaneously was subsequently converted another insoluble substance by condensation. The remainder of the soluble substances with amino-acids side by side in the solution with or without any chemical change in its structure.

¹ Bamberger and Berl : *Lieb. Ann.*, **275** 344(1893).

² Hoppe-Seyler: *Physiol. Pathol. Chem. Analyse*, 9 Aufl. 315 (1927); A. Kossel u. S. Edlbacher: *Z. physiol. Chem.*, **93**, 396 (1915); A. Windaus: *Ber. D. Chem. Ges.*, **43**, 499 (1910).

The study of the PH value of the individual reaction products and also their buffer action will be of some important service for the verification of the hypothesis with regard to the chemical reactions above mentioned, and the results will be described in detail in the next article.

October 1924. Laboratory of Organic- and Bio-Chemistry.

The Reversal of Lines in the Explosion-Spectrum of Lead.

(Continuation of the former report¹)

By

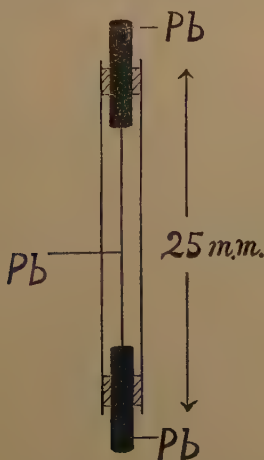
Bunsaku Arakatsu.

(Received June 26, 1926)

In the previous paper the writer reported the results on the self reversals of the spectral lines of lead appearing when a lead-wire is made to explode suddenly in the air of reduced pressure by passing a heavy electric current through it. Since then the writer has investigated the same phenomenon in the air of higher pressures.

As the explosion-chamber described in the former paper did not work well with the air of higher pressure, various preliminary devices were

tried, and at last the experiment were carried out with the following device shown in Fig. 1. A piece of thin lead wire to be exploded with a diameter of about 0.2mm. is stretched between two thick leading-in-wires of lead. This is inserted in a transparent quartz tube of various size, and the two leading-in-wires are sealed air tight by hard sealing wax or by soldering at the ends of the tube. As in the previous experiment, the lead wire is exploded by short circuiting suddenly the secondary circuit of 200 volts of a step-down-transformer, and the emitted light is observed by a small quartz spectrograph made by Hilger. When



1. These Memoirs, 10, 31(1926).

the sealing of the tube does not break, the spectrum lines show marked reversals in the intense continuous background as shown in the photograph reproduced in the Plate.

By keeping the diameter and the length of the wire to be exploded as constant the density of the vapour of lead formed by the explosion was made to vary by using enclosing quartz tube of various diameters, and the higher vapour pressure was attained by reducing the size of the quartz tube. When the enclosing quartz tube was 3 cms. in length and 5 mms. in diameter, the appearance of the explosion-spectrum showed no marked difference compared with that observed in the case when the wire was exploded in the open air.

But when tubes of the same length, and 3 mms., 2 mms. in diameter are used successively in this order the continuous background in the explosion-spectrum gets thicker and thicker and wider, extending even up to the region of the visible spectrum, and at the same time a remarkable change occurs among the emission and reversed lines; some emission and reversed lines which have previously appeared become weaker and weaker, while many reversal-lines get clearer and broader at the same time so that the whole spectrum becomes, at last, to consist of these reversal lines in the continuous background. The reversed lines which have thus survived up to this final stage are tabulated in the Table, and the characteristic behavior of each line is described below.

The five lines λ_{4057} ($2p_2-2s$), λ_{3740} , λ_{3684} ($2p_3-2s$), λ_{3640} ($2p_3-2s$) and λ_{3573} , which were difficult previously, though not impossible, to observe as reversed ones, become now broad and clear absorption lines. Of the lines relating to $2p_1$ which showed no reversal previously at all, the two lines λ_{4168} ($2p_1-3d_3$) and λ_{4020} ($2p_1-3d_1$) represent now weak reversal. The appearance of the reversed lines λ_{4057} ($2p_2-2s$), λ_{3684} ($2p_3-2s$), λ_{3640} ($2p_3-2s$) and λ_{2833} ($2p_1-2s$) are worthy of remark in that they diffuse toward the red side with the sharp violet side.

Among the lines belonging to the diffuse series the reversed lines λ_{2802} ($2p_2-3d_1$) and λ_{2614} ($2p_3-3d_2$), which were strong in emission, are broadened remarkably and symmetrically on both sides, while the other reversed lines of this series are tolerably sharp. When a drop of mercury is present in the explosion tube these peculiarities of the lines are still more enhanced as shown in the Plate. This will probably be due to the increased density of the ionized vapour.

Generally the aspects of the reversed lines are quite similar to those of their emission lines, and the reasoning given in the previous paper to explain the width of the absorption as well as of the emission lines by

the Stark-effect seems to be verified further by the present investigation.

Lastly, it seems worth while to mention the manner of development of the continuous background with increasing vapour pressure. With the explosion in a partially evacuated chamber a faint continuous band was detected in the region of $\lambda 2500$, and as the air pressure in the chamber increased this band became strong and wide by extending its region on both sides. Moreover, it seems probable that there is another centre of the continuous background in the extreme ultra-violet region of wave-lengths shorter than $\lambda 2240$, because the lines $\lambda 2253$, 2247 , etc. appeared, already in the stage of low pressure, as mere absorption lines in the continuous background. Finally, there is another continuous background whose centre lies between $\lambda 4058$ and $\lambda 3684$. This background was very weak at the stage of lowest pressure. As the pressure was increased this became stronger and wider and at the stage of highest pressure examined this was the most intense continuous background. Thus considering, though it may not be conclusive, it seems to be probable that the continuous spectrum has many centres of development. So far as the present experiment is concerned the development begins with that lying in the region of the shortest wave-length and then proceeds to the one lying in the region of longer wave-lengths successively as the vapour pressure is increased.

The writer expresses his sincere thanks to Prof. M. Kimura, who kindly interested himself in this experiment; the author's thanks are also due to Professor U. Yoshida, who kindly discussed the results of this work.

Table

Wave Length.	Series.	Nature of the Emission Lines.	Nature of the Absorption Lines.
4058	$2p_2 - 2s$	Broad. VS. RD.	The same as the emission
3740	————	D. RD.	Sharp. slightly RD.
3684	$2p_5 - 2s$	B. VS. RD.	The same as the emission
3640	$2p_3 - 2s$	B. VS. RD.	"
3573	————	D. RD.	S. RD.
2837	$2p_2 - 3d_3$	(D. ?)	S.
2833	$2p_4 - 2s$	S.	B. VS. RD.
2823	$2p_3 - 3d_2$	S.	S.
2802	$2p_2 - 3d_1$	B. D.	B. D.

Wave Length	Series.	Nature of the Emission Lines.	Nature of the Absorption Lines.
2663	$2p_2 - X_2$	S.	S. RD.
2657	$2p_3 - 3d_3$	D.	Faint. D.
2628	————	S.	Faint. S.
2614	$2p_3 - 3d_2$	B. D.	B. D.
2577	$2p_2 - X_1$	S.	S. RD.
2476	$2p_3 - X_2$	S. weak.	S. RD. weak.
2446	$2p_3 - 3^s$	S. weak.	S. faint.
2444	————	B. to both side.	S. faint.
2412	$2p_2 - 4d_3$	weak S.	S. faint.
2402	$2p_3 - X_1$		
2394	————		S.
2333	$2p_2 - 4^s$		
2247	$2p_3 - 4d_2$		
2238	$2p_3 - 4d_1$		

S. : Sharp.

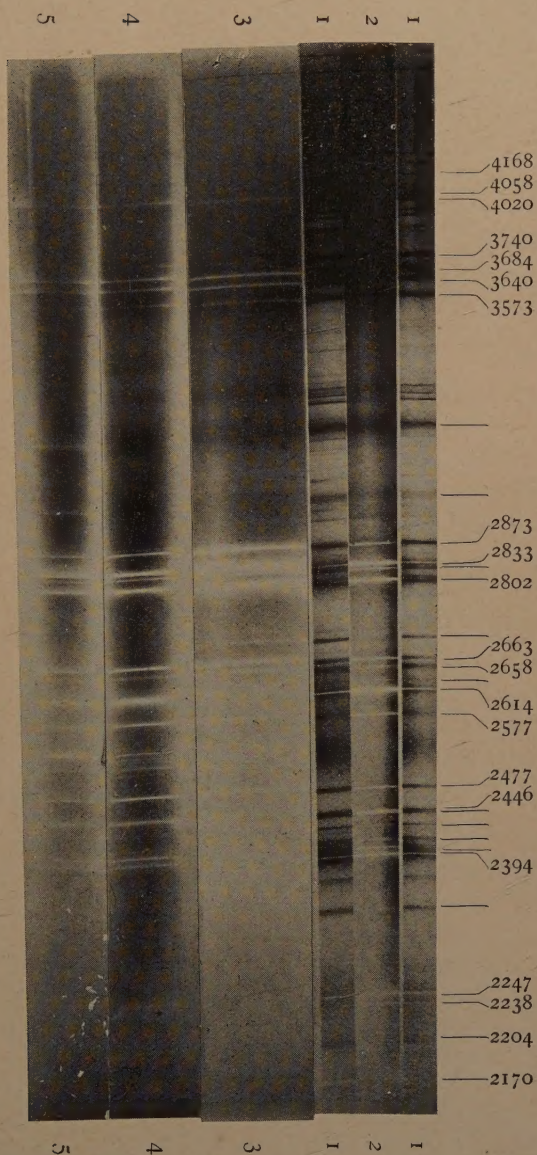
B. : Broad.

D. : Diffuse.

RD. : Diffuse toward red.

VS. : Sharp on violet side.

Reversal of Lines in the Explosion Spectrum of Lead.
(Negative Plate).



1. Explosion in open air.
2. Explosion in a closed tube.
3. Explosion in a closed tube.
4. Explosion in a closed tube at the presence of a drop of mercury.
5. Explosion in a closed tube at the presence of a drop of mercury.

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